A non-linear Beavers-Joseph interface condition derived from a kinetic energy balance

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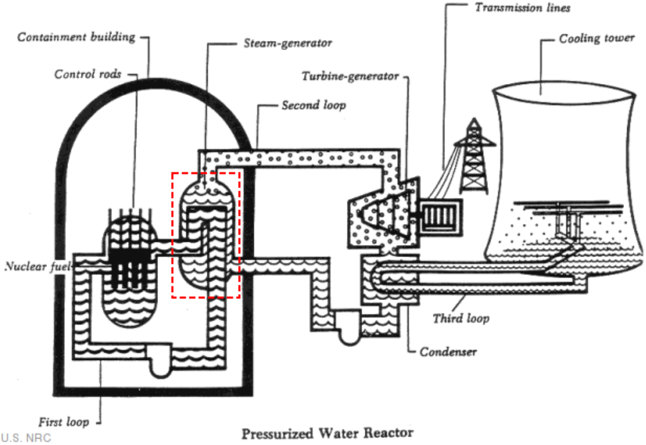
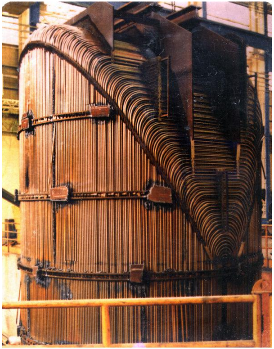
Pressurized Water Reactors (Generation II Nuclear Reactors)

Typical engineering problems for steam generators : at component scale design of steam generators (GENEPI code at CEA) ; at system scale coupling with core neutronics and thermohydraulics for safety analysis (CATHARE code at CEA)

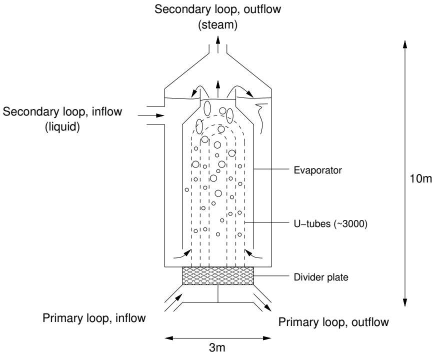


Pressurized Water Reactor

Steam Generators for Pressurized Water Reactors



U.S. NRC



Complex geometry and excessively fine scales involved in physics :

- ≈ 3000 U-tubes
- Tube support plates
- Anti-vibration bars

Steam Generators for Pressurized Water Reactors

GENEPI code physical models coupling primary loop (core) and secondary loop (steam generator) : homogeneous two-phase flow models with volume averaging over the porous medium represented by the *U-tubes*

- Primary fluid, energy balance :

$$\rho_P C_P \partial_t T_P + \rho_P C_P u_P \cdot \nabla T_P - \nabla \cdot (C_P \chi_T \nabla T_P) = -\frac{\gamma_0 h_{eq}}{\beta p_0}(T_P - T_W)$$

- Secondary fluid, mass balance

$$\beta \partial_t \rho + \nabla \cdot (\beta \rho u) = 0$$

- Secondary fluid, momentum balance

$$\beta \rho \partial_t u + \beta \rho u \cdot \nabla u - \mathrm{div}(\beta 2 \mu_T (\nabla u + \nabla^t u)) + \beta \nabla p = \beta \rho g - \beta \underline{\underline{\Lambda}} \cdot \rho u - \mathrm{div}(\beta x(1-x)\rho u_R \otimes u_R)$$

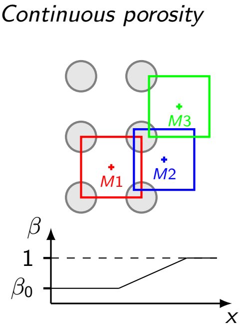
- Secondary fluid, energy balance

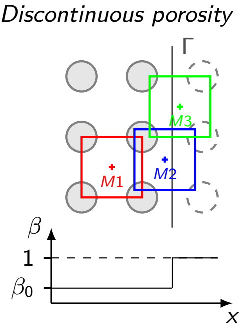
$$\beta \rho \partial_t H + \beta \rho u \cdot \nabla H - \nabla \cdot (\beta \chi_T \nabla H) = \tau \gamma_0 h_{eq}(T_P - T_W) - \nabla \cdot (\beta x(1-x)\rho \mathcal{L} u_R) + \cancel{Dp/Dt}$$

+ tabulated EOS for water

Underlying fluid-porous model ?

In practice : incoherent behaviour at fluid-porous interface (upper-part)





- Historically, GENEPI implements (w/ FEM) the *continuous porosity* approach. Mesoscopic scale ⇒ need to compute the transition region.
- Recent reflexions have favored the *discontinuous porosity* approach (w/ FVM). Macroscopic scale ⇒ transition modeled by transmission conditions.

Industrial problem vs Academic macroscopic models

- *Industrial problem* : inhomogeneous porous matrix (tube support plates, anti-vibration bars), complex flow model (compressible two-phase flow and coupling with primary loop), multi-dimensional flow w.r.t fluid-porous interface (Cheissoux & al., 1986).

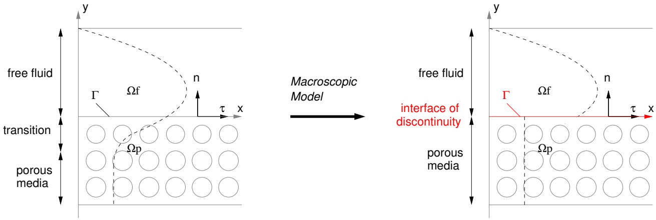
- *Models for the viscous regime* : correct macroscopic modelling ; mainly dedicated to homogeneous porous matrix, single-phase incompressible Stokes, flow parallel to the fluid-porous interface (Beavers–Joseph, 1967) (Ochoa-Tapia–Whitaker, 1995).

- *Towards the convective regime* : numerical studies show the dependence of Beavers-Joseph slip coefficient with the local Reynolds number at the interface (Sahraoui–Kaviany, 1992) (Liu–Prosperetti, 2011).
  Recent extension of Beavers-Joseph law through rigorous upscaling (Marciniak-Czochra–Mikelić, 2013).

- *Objective of this work* : existence of a generalized Beavers-Joseph law for the convective regime ? find well-posed interface condition for coupling Navier-Stokes/Forchheimer in the non-linear regime.

Classical fluid-porous macroscopic models

Permeability K, porosity ϕ, fluid viscosity $\tilde{\mu}^f = \mu$, effective viscosity $\tilde{\mu}^p = \mu/\phi$; u^f free-fluid velocity (local), u^p Darcy velocity (volume average).



Beavers–Joseph : Stokes/Darcy with slip velocity (1967)

$$-\mu\Delta u + \nabla p = f \text{ in } \Omega_f$$
$$\mu K^{-1} u + \nabla p = f \text{ in } \Omega_p$$
$$\nabla\cdot u = 0 \text{ in } \Omega_f \cup \Omega_p$$

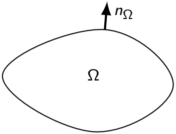
$$\boxed{\mu \partial_n u_\tau = \frac{\mu\alpha}{\sqrt{K}}(u^f_\tau - u^p_\tau)} \text{ on } \Gamma$$

Ochoa-Tapia–Whitaker : Stokes/Brinkman with stress jump (1995)

$$-\mu\Delta u + \nabla p = f \text{ in } \Omega_f$$
$$-\tilde{\mu}\Delta u + \mu K^{-1} u + \nabla p = f \text{ in } \Omega_p$$
$$\nabla\cdot u = 0 \text{ in } \Omega_f \cup \Omega_p$$

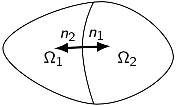
$$\boxed{\tilde{\mu}^f \partial_n u^f_\tau - \tilde{\mu}^p \partial_n u^p_\tau = \frac{\mu\beta}{\sqrt{K}} u^f_\tau} \text{ on } \Gamma$$

Multidomain formulation for the Stokes problem



Find $(u, p) \in \mathcal{H}^1(\Omega) \times \mathcal{L}^2(\Omega)$,

$$\begin{cases} -\mu \Delta u + \nabla p = 0 \text{ in } \Omega \\ \nabla \cdot u = 0 \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega \\ p = 0 \text{ on } \partial\Omega \end{cases}$$



$\Gamma = \overline{\Omega}_1 \cap \overline{\Omega}_2, \; n = n_1$

Find $(u_i, p_i) \in \mathcal{H}^1(\Omega_i) \times \mathcal{L}^2(\Omega_i)$,

$$\begin{cases} -\mu_i \Delta u_i + \nabla p_i = 0 \text{ in } \Omega_i \\ \nabla \cdot u_i = 0 \text{ in } \Omega_i \\ u_i = 0 \text{ on } \partial\Omega_i \cap \partial\Omega \\ p_i = 0 \text{ on } \partial\Omega_i \cap \partial\Omega \\ (\mu_i \nabla u_i - p_i I) \cdot n = (\mu_j \nabla u_j \\ \quad -p_j I) \cdot n \text{ on } \partial\Omega_i \cap \partial\Omega_j, \; j \neq i \end{cases}$$

Let $v \in \mathcal{H}^1(\Omega)$:

$$\int_\Omega -\mu \Delta u \cdot v + \int_\Omega \nabla p \cdot v = 0$$

$$\Leftrightarrow \int_{\partial\Omega} -\mu(\nabla u \cdot v) \cdot n_\Omega + \int_\Omega \mu \nabla u : \nabla v + \int_{\partial\Omega} p v \cdot n_\Omega - \int_\Omega p \nabla \cdot v = 0$$

$$\Leftrightarrow \sum_{i=1}^{2} \int_{\partial\Omega_i \cap \partial\Omega} -\mu_i(\nabla u_i \cdot v) \cdot n_i + \int_{\Omega_i} \mu_i \nabla u_i : \nabla v + \int_{\partial\Omega_i \cap \partial\Omega} p_i v \cdot n_i - \int_{\Omega_i} p_i \nabla \cdot v = 0$$

$$\Leftrightarrow \int_\Omega -\mu \Delta u \cdot v + \int_\Omega p \cdot v + \sum_{i=1}^{2} \int_{\partial\Omega_i \cap \Gamma} \mu_i (\nabla u_i \cdot v) \cdot n_i - \int_{\partial\Omega_i \cap \Gamma} p_i v \cdot n_i = 0$$

$$\Leftrightarrow \boxed{[\![\mu \nabla u - pI]\!]_\Gamma \cdot n = 0 \quad \text{with } [\![x]\!]_\Gamma = x^2|_\Gamma - x^1|_\Gamma}$$

General JEBC mathematical model for Stokes

Jump Embedded Boundary Conditions (JEBC) : mathematical tool introduced in (Angot, 2003) for modeling and analyzing problems featuring interfaces of discontinuity, inferring fictitious domains and embedded boundary conditions.

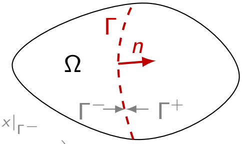
Jump model, derived from the multidomain formulation :

Find $(u, p) \in \mathcal{H}_0^1(\Omega) \times \mathcal{L}^2(\Omega)$,

$$\begin{cases} -\mu \Delta u + \nabla p = 0 \text{ in } \Omega \\ \nabla \cdot u = 0 \text{ in } \Omega \\ [\![(\mu \nabla u - pI) \cdot n]\!]_\Gamma = M \bar{u}|_\Gamma - h \text{ on } \Gamma \\ \overline{(\mu \nabla u - pI) \cdot n}|_\Gamma = S [\![u]\!]|_\Gamma - g \text{ on } \Gamma \end{cases}$$

$$[\![x]\!]_\Gamma = x|_{\Gamma+} - x|_{\Gamma-}$$

$$\bar{x}|_\Gamma = \frac{1}{2}\left(x|_{\Gamma+} + x|_{\Gamma-}\right)$$



Intended to represent an immersed interface between independent subdomains or transmission conditions between coupled subdomains.

Example of coefficients pairs (Angot, 2005) :

- $M = 4S = 2\alpha_R I$ and $h/2 - g = g_R \Rightarrow (\mu \nabla u - pI)|_{\Gamma-} \cdot n = \alpha_R u|_{\Gamma-} - g_R$
- $M = 4S = 2/\varepsilon I$ and $h/2 - g = u_D/\varepsilon$ with $\varepsilon \to 0 \Rightarrow u|_{\Gamma-} = u_D$

Well-posedness proved in (Angot, 2010) for the general JEBC jump model.

Application to macroscopic fluid-porous models

Coefficient pairs with $g = h = 0$ (Angot, 2011) :

Beavers–Joseph

$$M = \varepsilon \tau \otimes \tau + \frac{\mu \beta_n}{\sqrt{K_n}} n \otimes n$$

$$S = \frac{\mu \alpha_\tau}{\sqrt{K_\tau}} \tau \otimes \tau + \frac{1}{\varepsilon} n \otimes n$$

Ochoa-Tapia–Whitaker

$$M = \frac{\tilde{\mu} \beta_\tau}{\sqrt{K_\tau}} \tau \otimes \tau + \frac{\tilde{\mu} \beta_n}{\sqrt{K_n}} n \otimes n$$

$$S = \frac{1}{\varepsilon} I$$

Embedded jump conditions in the limit $\varepsilon \to 0$:

Beavers–Joseph

$$[\![\mu \partial_n u_\tau]\!]_\Gamma = 0$$

$$[\![u_n]\!]_\Gamma = 0$$

$$[\![-\mu \partial_n u_n + p]\!]_\Gamma = \frac{\mu \beta_n}{\sqrt{K_n}} \overline{u_n}|_\Gamma$$

$$\overline{(-\mu \partial_n u_\tau)}|_\Gamma = \frac{\mu \alpha_\tau}{\sqrt{K_\tau}} [\![u_\tau]\!]_\Gamma$$

Ochoa-Tapia–Whitaker

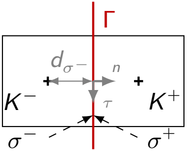
$$[\![-\tilde{\mu} \partial_n u_\tau]\!]_\Gamma = \frac{\tilde{\mu} \beta_\tau}{\sqrt{K_\tau}} \overline{u_\tau}|_\Gamma$$

$$[\![u_n]\!]_\Gamma = 0$$

$$[\![-\mu \partial_n u_n + p]\!]_\Gamma = \frac{\mu \beta_n}{\sqrt{K_n}} \overline{u_n}|_\Gamma$$

$$[\![u_\tau]\!]_\Gamma = 0$$

Cell-centered Finite-Volume discretization



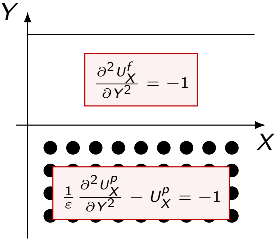
$$(\partial_n u)_\sigma = (u_\sigma - u_K)/d_\sigma$$
$$[\![u]\!]_\sigma = u_{\sigma^-} - u_{\sigma^+}$$
$$\bar{u}|_\sigma = (u_{\sigma^-} + u_{\sigma^+})/2$$

$$\begin{pmatrix} \frac{\mu_{\sigma-}}{d_{\sigma-}} - \frac{M_{\tau\tau}}{2} & \frac{\mu_{\sigma+}}{d_{\sigma+}} - \frac{M_{\tau\tau}}{2} & -\frac{M_{\tau n}}{2} & -\frac{M_{\tau n}}{2} & 0 & 0 \\ \frac{\mu_{\sigma-}}{2d_{\sigma-}} - S_{\tau\tau} & -\frac{\mu_{\sigma+}}{2d_{\sigma+}} + S_{\tau\tau} & -S_{\tau n} & -S_{\tau n} & 0 & 0 \\ -\frac{M_{n\tau}}{2} & -\frac{M_{n\tau}}{2} & \frac{\mu_{\sigma-}}{d_{\sigma-}} - \frac{M_{nn}}{2} & \frac{\mu_{\sigma+}}{d_{\sigma+}} - \frac{M_{nn}}{2} & 1 & -1 \\ -S_{n\tau} & S_{n\tau} & \frac{\mu_{\sigma-}}{2d_{\sigma-}} - S_{nn} & -\frac{\mu_{\sigma+}}{2d_{\sigma+}} + S_{nn} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \times \begin{pmatrix} (u_\tau)_{\sigma^-} \\ (u_\tau)_{\sigma^+} \\ (u_n)_{\sigma^-} \\ (u_n)_{\sigma^+} \\ p_{\sigma^-} \\ p_{\sigma^+} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\mu_{\sigma-}}{d_{\sigma-}} & \frac{\mu_{\sigma+}}{d_{\sigma+}} & 0 & 0 \\ \frac{\mu_{\sigma-}}{2d_{\sigma-}} & -\frac{\mu_{\sigma+}}{2d_{\sigma+}} & 0 & 0 \\ 0 & 0 & \frac{\mu_{\sigma-}}{d_{\sigma-}} & \frac{\mu_{\sigma+}}{d_{\sigma+}} \\ 0 & 0 & \frac{\mu_{\sigma-}}{2d_{\sigma-}} & -\frac{\mu_{\sigma+}}{2d_{\sigma+}} \end{pmatrix} \times \begin{pmatrix} (u_\tau)_{K^-} \\ (u_\tau)_{K^+} \\ (u_n)_{K^-} \\ (u_n)_{K^+} \end{pmatrix} - \begin{pmatrix} h_\tau \\ g_\tau \\ h_n \\ g_n \end{pmatrix}$$

Numerical example

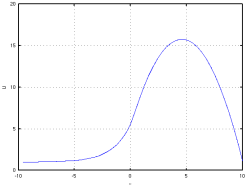
Analytic verification test from (Ochoa-Tapia–Whitaker, 1995). Assumptions : pressure driven flow, steady-state, flow parallel to the fluid-porous interface.



$$\frac{\partial^2 U_X^f}{\partial Y^2} = -1$$

$$\frac{1}{\varepsilon}\frac{\partial^2 U_X^p}{\partial Y^2} - U_X^p = -1$$

$$U_X^f\Big|_\Gamma = U_X^p\Big|_\Gamma$$
$$\frac{1}{\phi}\frac{\partial U_X^p}{\partial Y}\Bigg|_\Gamma - \frac{\partial U_X^f}{\partial Y}\Bigg|_\Gamma = \beta U_X|_\Gamma$$



Horizontal velocity profile obtained using JEBC conditions expressed as internal boundary conditions on the horizontal velocity on Γ.

A jump model devised from a kinetic energy balance

Coupling Navier-Stokes/Forchheimer ?

- On the physics side : discontinuity of u_τ should yield to a jump of the kinetic energy at the interface
- On the maths side : in a previous work (Angot, 2013) proof of well posedness of the Stokes problem with a non-linear interface condition

We attempt an extension of Beavers-Joseph law involving a jump of the non-linear term at the interface :

$$\mu \partial_n u_\tau^f = \underbrace{\frac{\mu \alpha_{\text{kin}}}{2|u_\tau^f|\sqrt{K}}(|u_\tau^f|^2 - |u_\tau^p|^2)}_{\text{kinetic energy jump at interface}}$$

Version 1

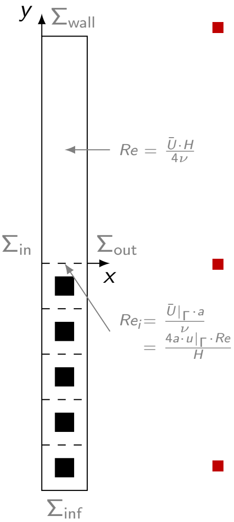
$$\mu \partial_n u_\tau^f = \frac{\mu \alpha_{nl}}{\sqrt{K}}(u_\tau^f - u_\tau^p)$$
$$\text{with } \alpha_{nl} = \frac{\alpha_{\text{kin}}}{2}\left(1 + \frac{u_\tau^p}{u_\tau^f}\right)$$

Version 2, parameter $\gamma(\phi) \geq 1$

$$\mu \partial_n u_\tau^f = \frac{\mu \alpha_{nl}}{\sqrt{K}}(u_\tau^f - u_\tau^p)$$
$$\text{with } \alpha_{nl} = \frac{\alpha_{\text{kin}}}{2}\left(1 + \frac{u_\tau^p}{u_\tau^f}\right)^\gamma$$

Moreover for consitency with the viscous regime : $\lim_{Re \to 0} \alpha_{nl} = \alpha_{BJ}$

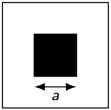
Direct Numerical Simulation : problem definition



- *Non-dimensional problem* : steady-state Navier-Stokes equations on $\Omega = [-H/2, H/2] \times [0, L]$ with $H = 4$ and $L = 0.4$

$$\begin{cases} \nabla \cdot (u \otimes u) - \dfrac{1}{Re} \Delta u + \nabla \tilde{p} + \underbrace{< \nabla p >}_{= 1 \cdot e_x} = 0 & \text{in } \Omega \\ \nabla \cdot u = 0 & \text{in } \Omega \\ u|_{\Sigma_{\text{in}}} = u|_{\Sigma_{\text{out}}}, \ \ \tilde{p}|_{\Sigma_{\text{in}}} = \tilde{p}|_{\Sigma_{\text{out}}} & \text{on } \partial\Omega \\ \partial_n u_x|_{\Sigma_{\text{inf}}} = 0, \ \ u_y|_{\Sigma_{\text{inf}}} = 0, \ \ u|_{\Sigma_{\text{wall}}} = 0 & \end{cases}$$

- *Porous inclusions* : for a unit periodic cell w/ square inclusion,



a	ϕ	K
2/8	0.94	6.0e-2
4/8	0.75	1.4e-2
6/8	0.44	1.6e-3
7/8	0.23	2.1e-4

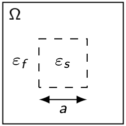
- *Numerical method* : unsteady Navier-Stokes equations solved using a 2$^{\text{nd}}$ order pressure-correction scheme with cell-centered finite-volumes on a uniform Cartesian grid 64×640

DNS : post-processing the permeability

- *Local velocity* : Stokes problem on $\Omega = [0,1] \times [0,1]$ with square inclusion

$$\begin{cases} -\mu \Delta u + \nabla \tilde{p} + \underbrace{<\nabla p>}_{\text{given}} = \frac{1}{\varepsilon} u \text{ in } \Omega \\ \nabla \cdot u = 0 \text{ in } \Omega \\ u, \tilde{p} \text{ periodic on } \partial\Omega \end{cases}$$

$\mu = 1$



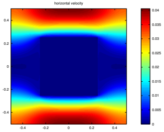
$\varepsilon_s \to 0$
$\varepsilon_f \to \infty$

- *Upscaling* : Darcy's law

$$<u> = -\frac{1}{\mu} \begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} \underbrace{<\nabla p>}_{=1 \cdot e_x} \Rightarrow K_{xx} = -\mu <u> \cdot e_x$$

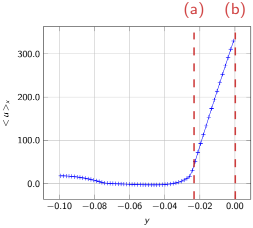
- *Results* : symmetry $\Rightarrow K_{xx} = K_{yy}$ and $K_{xy} = 0$

Grid size	$a = 2/8$	$a = 4/8$	$a = 6/8$	$a = 7/8$
32×32	0.067500	0.016404	0.0021873	3.3775e-04
64×64	0.063030	0.015031	0.0018600	2.6829e-04
128×128	0.060865	0.014361	0.0017057	2.2860e-04
256×256	0.059797	0.014029	0.0016308	2.0987e-04



DNS : post-processing the normal derivative on Γ

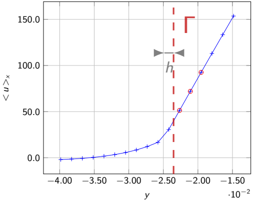
- *Interface position* : what relevant choice for interface of discontinuity w.r.t free-fluid ? widely discussed in litterature but no clear optimal choice yet



(a) Γ tangent to inclusion, shifted by h
(b) Γ tangent to porous cell

Our choice : (a) so as to avoid considering a too large free-fluid region as belonging to the transition region

- *Normal derivative* : which stencil and which order ?

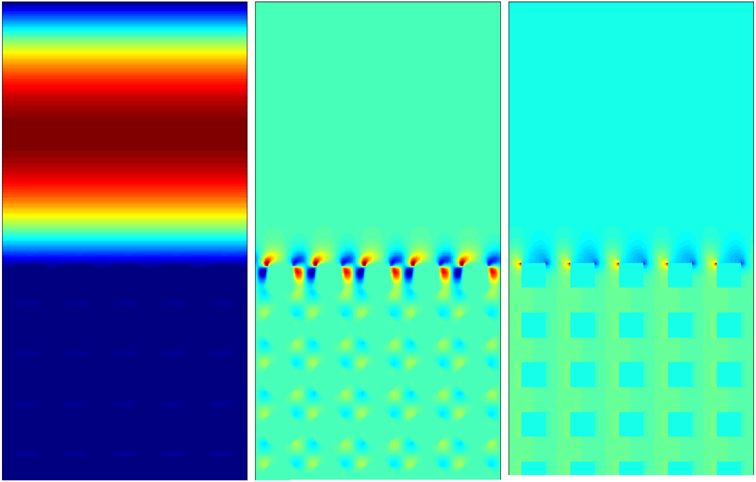


(a) Three point finite difference across Γ
(b) Three point finite difference at right of Γ

Our choice : (b) in order to discard completely the transition region

DNS : numerical results (1)

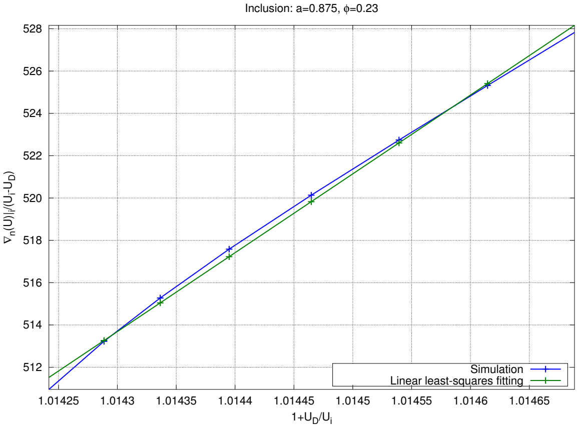
Velocity and pressure fields :



horizontal velocity vertical velocity pressure

DNS : numerical results (2)

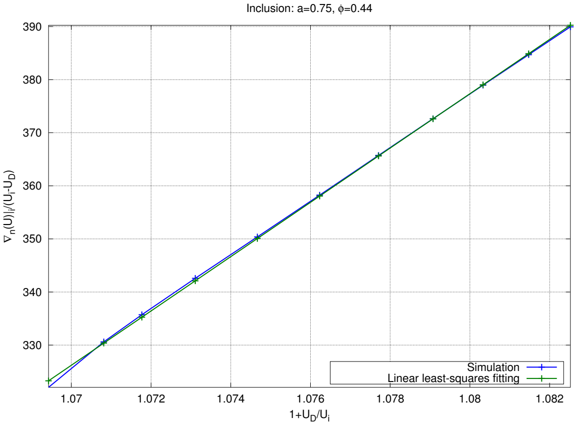
Fitting of interface model, version 1



Re	Re_i	u_i	u_D
10	7.0e-2	2.0e-2	2.9e-4
20	2.8e-1	4.0e-2	5.7e-4
30	6.3e-1	6.0e-2	8.6e-4
40	1.1	7.9e-2	1.1e-3
50	1.7	9.9e-2	1.4e-3
60	2.5	1.2e-1	1.7e-3
70	3.3	1.4e-1	2.0e-3
80	4.3	1.6e-1	2.3e-3

DNS : numerical results (2)

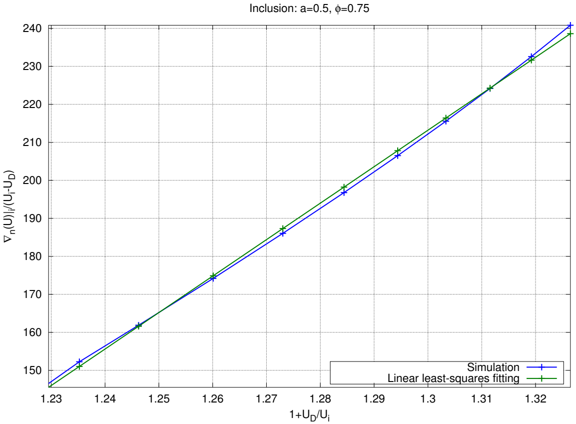
Fitting of interface model, version 1



Re	Re_i	u_i	u_D
1	1.0e-3	3.4e-3	2.4e-4
10	1.0e-1	3.3e-2	2.4e-3
20	3.9e-1	6.6e-2	4.7e-3
30	8.7e-1	9.7e-2	7.1e-3
40	1.5	1.3e-1	9.4e-3
50	2.3	1.5e-1	1.2e-2
60	3.3	1.8e-1	1.4e-2
70	4.4	2.1e-1	1.6e-2
80	5.6	2.3e-1	1.9e-2
90	7.0	2.6e-1	2.1e-2
100	8.6	2.9e-1	2.4e-2

DNS : numerical results (2)

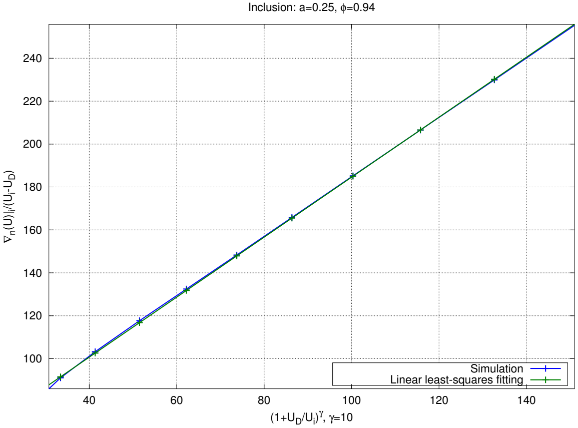
Fitting of interface model, version 1



Re	Re_i	u_i	u_D
1	1.9e-3	9.3e-3	2.1e-3
10	1.8e-1	9.0e-2	2.1e-2
20	6.9e-1	1.7e-1	4.2e-2
30	1.5	2.4e-1	6.3e-2
40	2.5	3.1e-1	8.4e-2
50	3.7	3.7e-1	1.1e-1
60	5.1	4.3e-1	1.3e-1
70	6.8	4.8e-1	1.5e-1
80	8.6	5.4e-1	1.7e-1
90	11	5.9e-1	1.9e-1
100	13	6.4e-1	2.1e-1

DNS : numerical results (3)

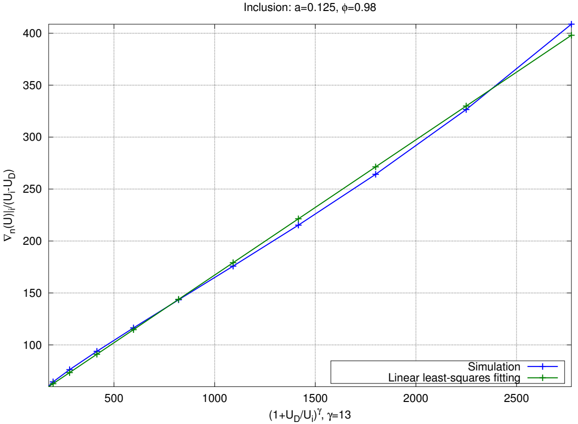
Fitting of interface model, version 2 with parameter γ



Re	Re_i	u_i	u_D
1	2.1e-3	2.1e-2	8.6e-3
10	2.0e-1	2.0e-1	8.5e-2
20	7.5e-1	3.8e-1	1.7e-1
30	1.6	5.2e-1	2.5e-1
40	2.6	6.5e-1	3.3e-1
50	3.9	7.7e-1	4.1e-1
60	5.2	8.7e-1	4.9e-1
70	6.7	9.6e-1	5.6e-1
80	8.3	1.0	6.3e-1
90	10	1.1	7.0e-1
100	12	1.2	7.7e-1

DNS : numerical results (3)

Fitting of interface model, version 2 with parameter γ



Re	Re_i	u_i	u_D
1	1.7e-3	3.5e-2	1.7e-2
10	1.7e-1	3.3e-1	1.7e-1
20	6.1e-1	6.1e-1	3.3e-1
30	1.2	8.3e-1	4.9e-1
40	2.0	1.0	6.4e-1
50	2.9	1.1	7.7e-1
60	3.8	1.3	9.0e-1
70	4.8	1.4	1.0
80	5.9	1.5	1.1
90	7.0	1.6	1.3
100	8.2	1.6	1.4

DNS : conclusion

The second model is necessary to match cases with high porosities :

$$\mu \partial_n u_\tau^f = \frac{\mu \alpha_{nl}}{\sqrt{K}} (u_\tau^f - u_\tau^p)$$
$$\text{with } \alpha_{nl} = \frac{\alpha_{\text{kin}}}{2} \left(1 + \frac{u_\tau^p}{u_\tau^f}\right)^{\gamma}$$

assuming :

- $\gamma = 1$ for $\phi < 0.75$
- $\gamma \geq 1$ for $\phi \geq 0.75$

ϕ	γ
0.75	1
0.94	10
0.98	13

Should rather consider an extension of Ochoa-Tapia–Whitaker law instead of Beavers–Joseph law for high porosities ?

Results and open problems

Current results :

- JEBC applicable to classical interface models
- Macroscopic interface condition for the non-linear regime
- Verified for highly convective flows

Future work :

- Proper JEBC model generalizing our interface condition
- Investigate the pressure jump at interface for “true” 2D flows
- DNS with compressible, two phase-phase flow

Questions ?

References

1 P. Angot (2003), *A model of fracture for elliptic problems with flux and solution jumps*, C.R. Acad. Sci. – Ser. I, vol. 337, pp. 425–430.

2 P. Angot (2010), *A fictitious domain model for the Stokes/Brinkman problem with jump embedded boundary conditions*, C.R. Acad. Sci. – Ser. I, vol. 348, pp. 697–702.

3 P. Angot (2011), *On the well-posed coupling between free fluid and porous viscous flows*, Applied Mathematics Letters, vol. 24, pp. 803–810.

4 P. Angot (2013), *On the unsteady Stokes problem with a nonlinear open artificial boundary condition modelling a singular load*, in preparation (*http://hal.archives-ouvertes.fr/hal-00820404*).

5 G.S. Beavers and D.D. Joseph (1967), *Boundary conditions at a naturally permeable wall*, Journal of Fluid Mechanics, vol. 30, pp. 197–207.

6 T. Carraro, C. Goll, A. Marciniak-Czochra and A. Mikelić (2013), *Pressure jump interface law for the Stokes-Darcy coupling : confirmation by direct numerical simulations*, Journal of Fluid Mechanics, vol. 732, pp. 510–536.

7 J.L. Cheissoux, J.F. Haquet, M. Grandotto, M. Bernard and E. de Langre (1986), *Spécifications physiques du code GENEPI (Générateurs de vapeur des REP)*, Technical Report, CEA/DRE/STRE/LGV, 86/756.

8 Q. Liu and A. Prosperetti (2011), *Pressure-driven flow in a channel with porous walls*, Journal of Fluid Mechanics, vol. 679, pp. 77–100.

9 A. Marciniak-Czochra and A. Mikelić (2013), *A nonlinear effective slip interface law for transport phenomena between a fracture flow and a porous medium*, Discrete and Continuous Dynamical Systems Series S, vol. 7, pp. 1065–1077.

10 J.A. Ochoa-Tapia and S. Whitaker (1995), *Momentum transfer at the boundary between a porous medium and a homogeneous fluid I – Theoretical development*, Int. J. Heat Mass Transfer., vol. 38, pp. 2635–2646.

11 J.A. Ochoa-Tapia and S. Whitaker (1995), *Momentum transfer at the boundary between a porous medium and a homogeneous fluid II – Comparison with experiment*, Int. J. Heat Mass Transfer., vol. 38, pp. 2647–2655.

12 M. Sahraoui and M. Kaviany (1992), *Slip and no-slip velocity boundary conditions at interface of porous, plain media*, Int. J. Heat Mass Transfer, vol. 35, pp. 927–943.