Event-based numerical simulation of slightly compressible two-phase flow in heterogeneous porous media applied to CO_2 injection in saline aquifers

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Presentation Outline

1. Motivation.
2. Event-based Methodology (DES)
3. Implementation for Incompressible Flow
4. Implementation for Slightly Compressible flow
5. Results. Comparison between TDS and DES simulation processes for Slightly Compressible two-phase flow.
6. Summary and work in progress.

Motivation

To develop a composite methodology that allows for improved performance in reservoir simulations containing (at least) the following three main characteristics:

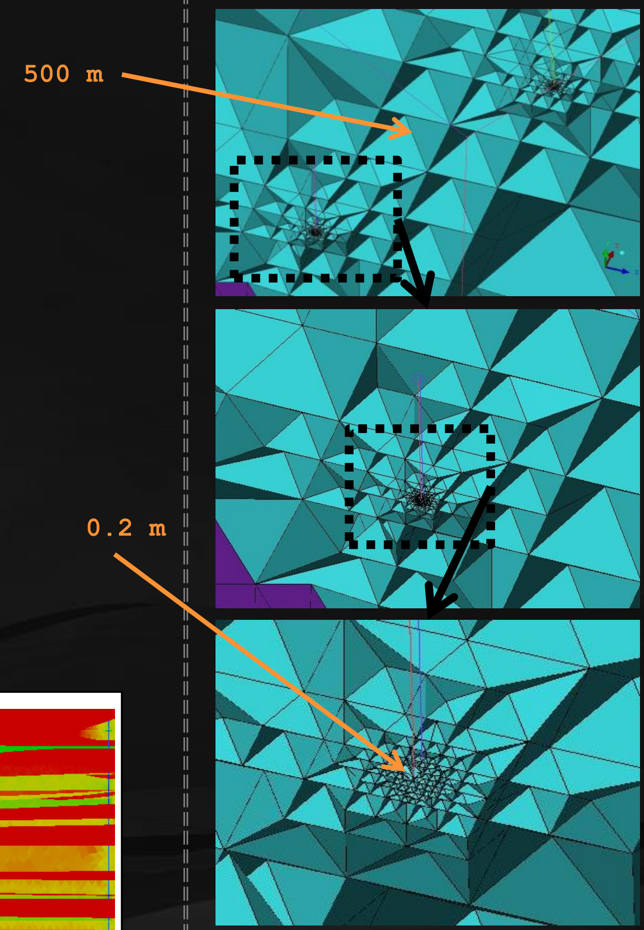
1. **Material heterogeneity:** causes large discrepancy in CFL requirements accross the computational domain.
2. **Multi-scale resolution:** detailed geometrical models are needed for greater accuracy and predictive capabilities.
3. **Localized phenomena**: such as evolving fronts, and other non-linear phenomena.

In summary: the targets are those simulations where "a lot is going on" in a relatively small portion of the domain.

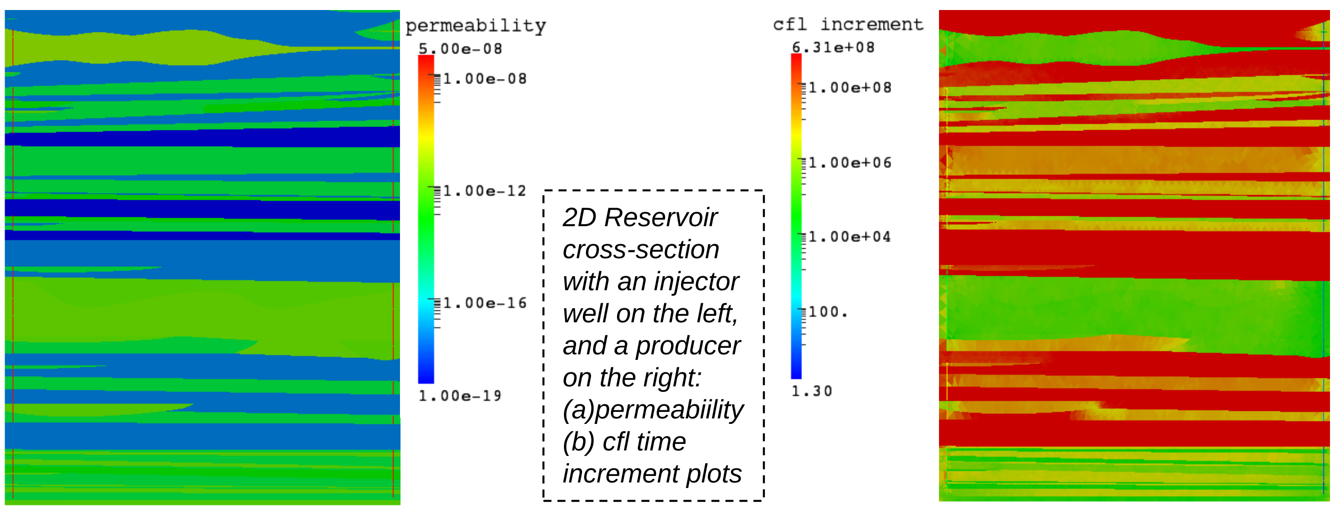
Without the loss of general applicability, this method is intended for modelling *CO_2 injection in saline aquifers*.

Motivation

- Grid-Based Timestep-Driven Simulation (TDS) schemes:
  - Cells evolve **synchronously** (i.e. global time stepping)
  - Courant–Friedrichs–Lewy[1] (Courant et al, 1928) condition.
    - **Explicit solution methods (ETDS)**:
      - Conditionally stable. Unbounded error propagation can be triggered by localized overstepping.
    - **Implicit solution methods (ITDS)**:
      - Unconditionally stable. Convergence with large "oversteps" is difficult, particularly with strong influence of sources (wells) and gravity effects on the velocity field.
  - Large variations in CFL conditions (heterogeneity + multi-scale resolution + non-linearities)

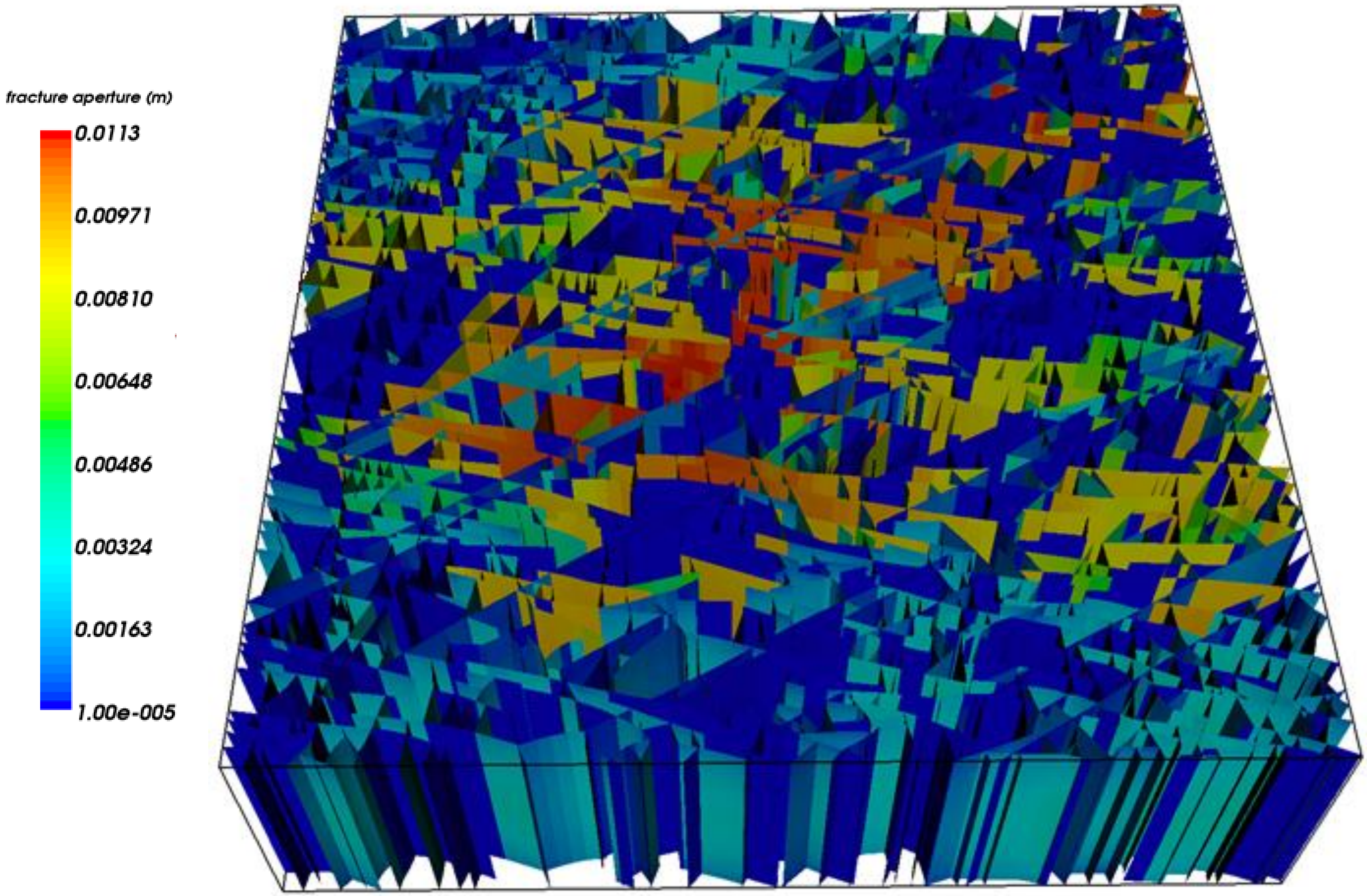


Zooming in from top to bottom, on a mesh cut-plane of 3D simplified 9x9x6 km discretized model, including an injector-producer pair. (Non-smoothed mesh)



2D Reservoir cross-section with an injector well on the left, and a producer on the right: (a)permeabiility (b) cfl time increment plots

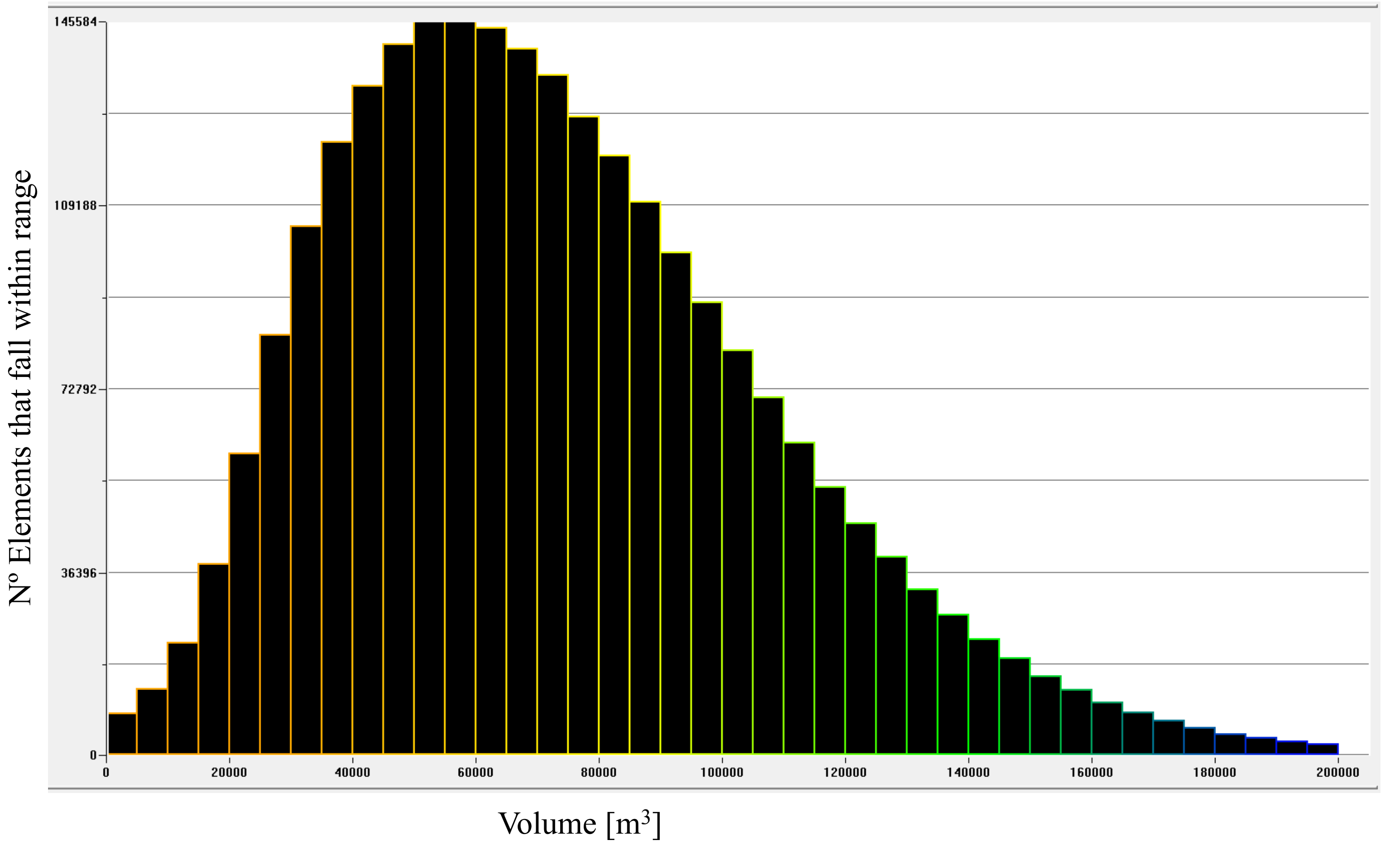
Motivation



fracture aperture (m)
0.0113
0.00971
0.00810
0.00648
0.00486
0.00324
0.00163
1.00e-005

Motivation

- A reservoir simulation model containing roughly 2.5 million cells contains the following distribution of sizes (volume).



- It is also very likely that extremely small elements with large velocities exist near wells, thus imposing restrictions on timestepping (explicit) or time accuracy (implicit).

Event-based Methodology

- Event-based or Discrete Event Simulation (DES) Methodology has its **roots in management science**
- It is based on the **chronological** and **conditional triggering** of a sequence of events.
- First introduced into the simulation process for PDE's by Omelchenko and Karimabadi (Olmechenko and Karimabadi, 2006), with application to Plasma Physics.
- Linked to DEVS (Discrete Event System Specification) formalism invented by (Zeigler B. P.,1976)
- Similar methodologies applied to ODE's where introduced (among others) into:
    - Gas dynamics (Nutaro, 2003)
    - Non-linear Elasto-dynamics (Lew et al, 2003)

Event Priority Queue

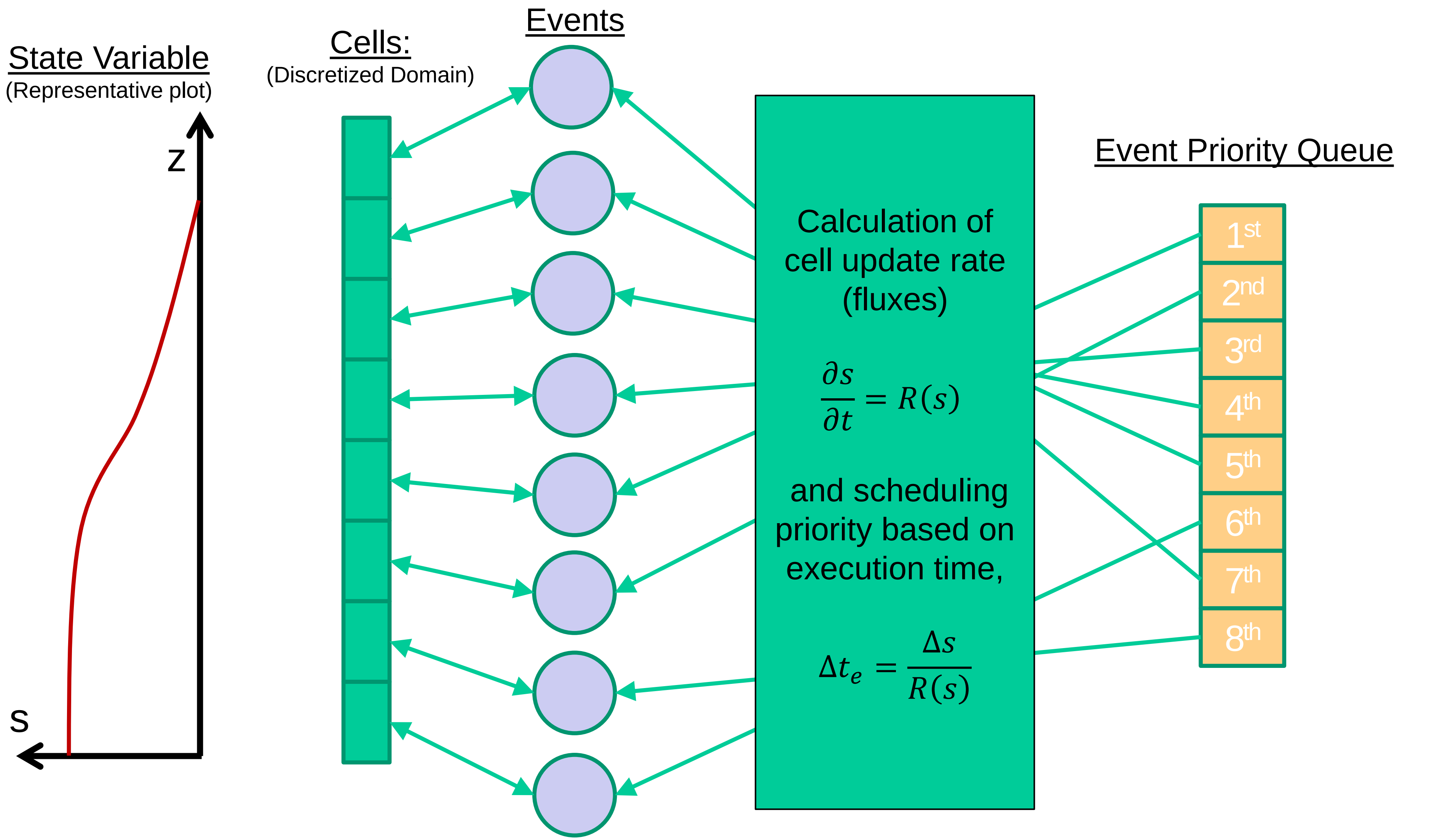
1st
2nd
3rd
4th
5th
6th
7th
8th

Event-based Methodology

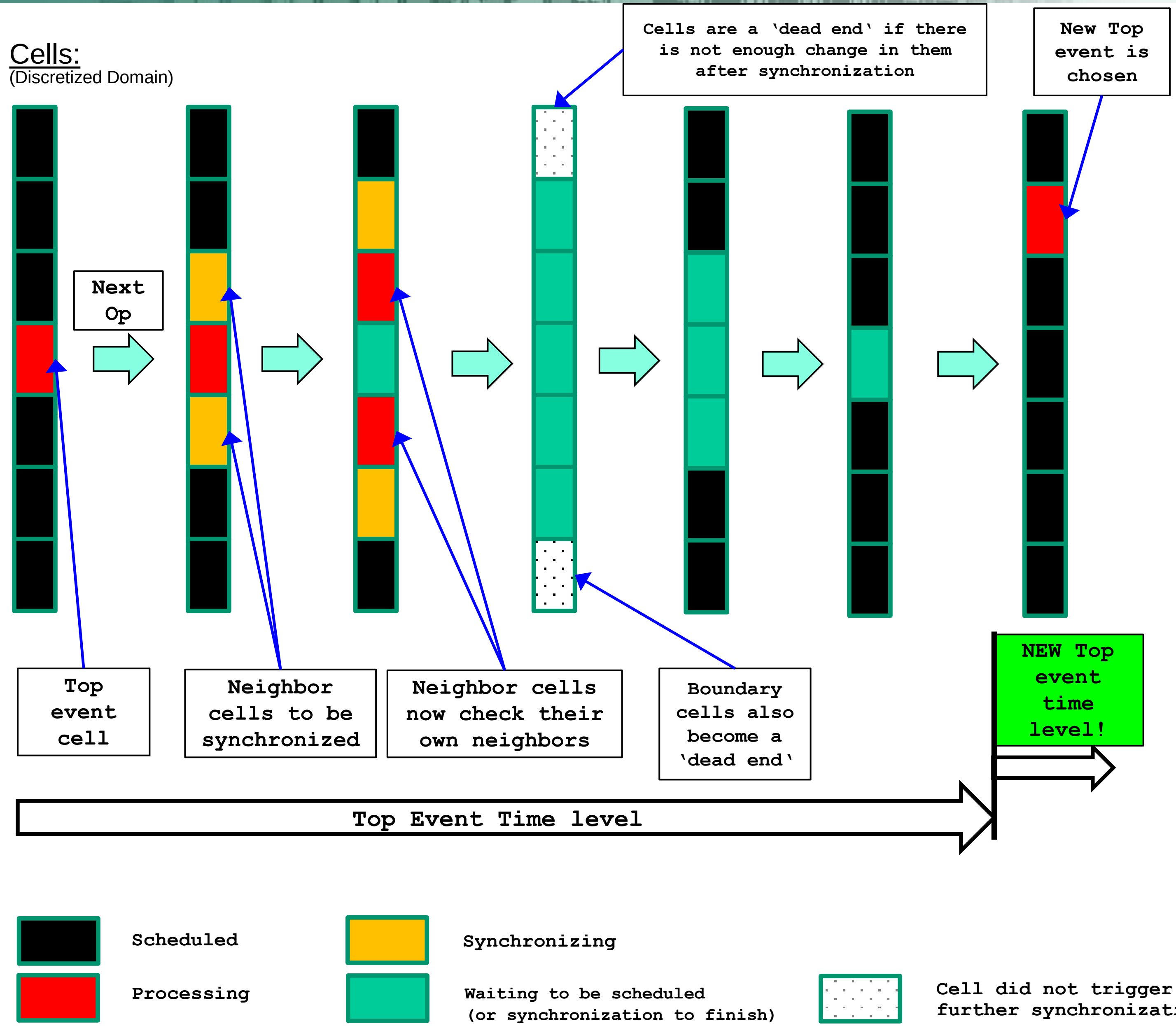
- Key implementation aspects for continuous systems (described by PDEs discretized by, *for example, Finite Volumes*):
  - DES is an essentially ***<u>explicit scheme</u>***.
  - In contrast to **ETDS** and **ITDS** schemes, cells evolve **<u>asynchronously</u>**.
  - Every change of a state variable in a grid cell is a possible **<u>“event”</u>** which is prioritized in a queue of events to be executed at a particular time. (<u>earliest always comes first</u>!)
  - **<u>Conservation</u>** is ensured through a flux synchronization process on both cells that share a face.
  - Synchronization may cause “preemptive” processing of a neighbour event (linked to the neighbour cell) to **satisfy causality constraints**.

Event-based Methodology

- DES queue assembly procedure. (vertical 1D problem example)



Event-based Methodology



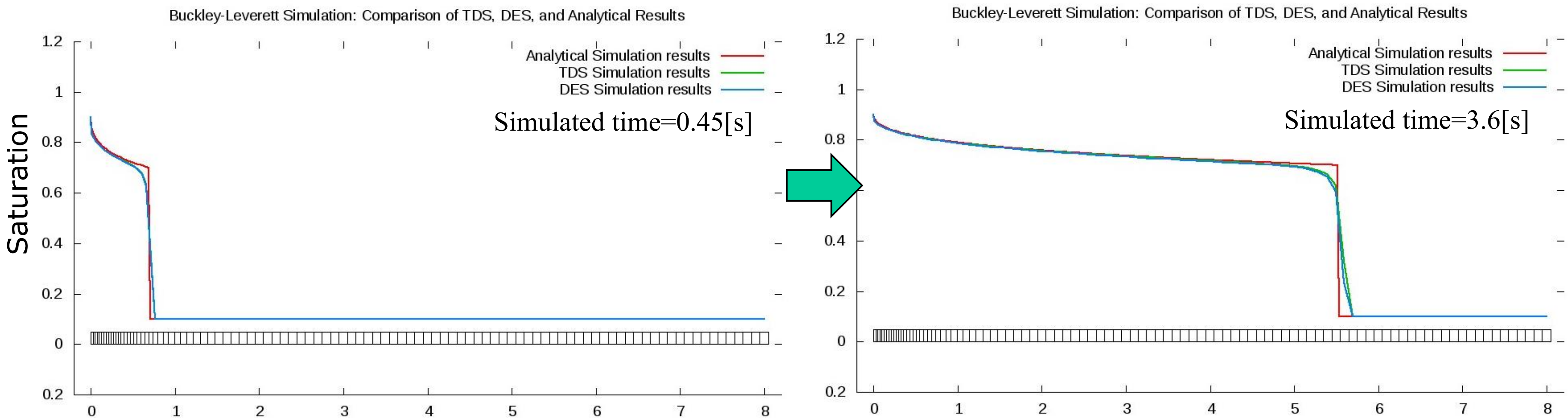
- So what happens during the DES step?
  - Events are processed beginning at the "Top" event given by the priority queue.
  - Synchronization with surrounding cells/events might trigger earlier processing of those synchronized cells/events.
  - The update that the top event started, ends once there are no more cells to synchronize and the original top event is scheduled
  - The next top event in line is chosen <u>and the simulation clock is updated</u>.

Event-based Methodology

- The DES algorithmic paradigm is a promising method because :
  - **Conservation** is ensured through a flux synchronization process on both sides of the cell that share a face.
  - **Adaptive synchronization** allows for a need-basis state variable updates, ensuring causality.
  - **It is a robust scheme**. CFL requirements can be maintained locally, and ensured via a “self adaptive” order of execution.

Implementation for incompressible two phase flow

- **<u>Assumptions/Simplications:</u>** Incompressible, two-phase, immiscible flow in the absence of gravity and capillary forces. Brooks-Corey relative permeability model.
- Constant injection rate from the left boundary.
- 1rst order upwinding advection scheme.

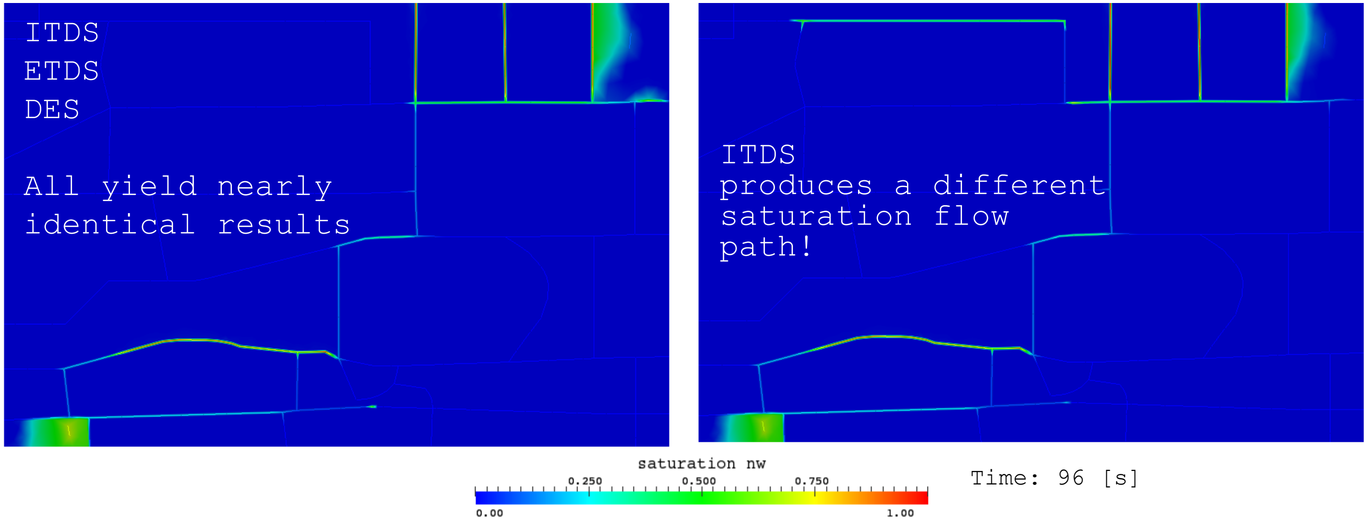


Min/max element size ratio*	Number of Cells	DES CPU Time [s]	ETDS CPU Time [s]	Speedup %
1/10	59	0.79	0.89	11.2
1/20	70	1.18	1.96	39.8
1/30	78	1.58	3.14	49.7
1/40	85	2.01	4.62	56.5
1/50	91	2.54	6.17	58.8

*Reference max. element size = 0.2 m

Implementation for incompressible two phase flow

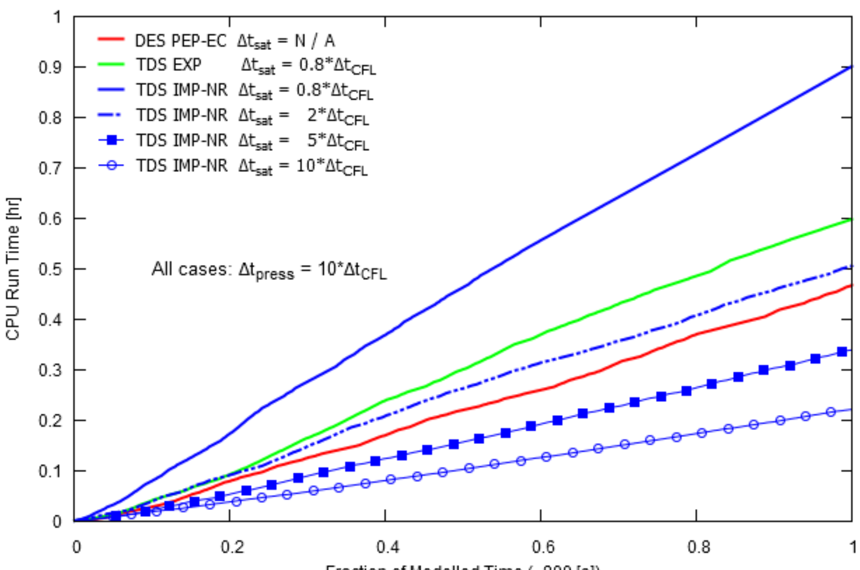
WITHOUT Saturation CFL Overstepping (0.8*CFL) | WITH Saturation CFL Overstepping (10*CFL)



Simulation Characteristics

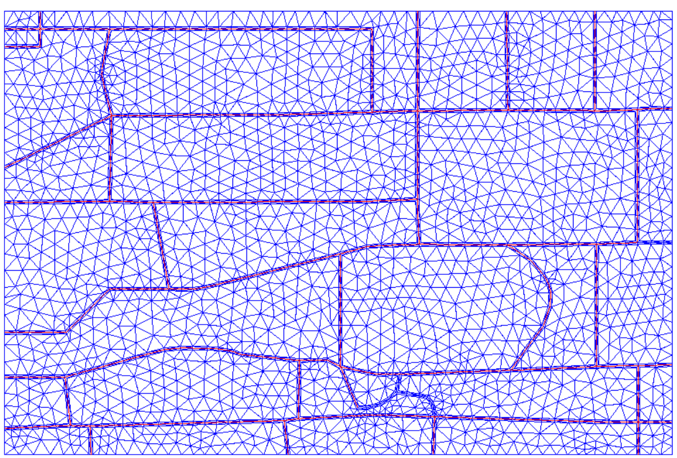
- K_m=2.0 e -14 K_f=8.3 e -8
- Brooks Corey Rel. Perm. model.
- **Injector (nw) well at lower left corner.**
  Producer well at upper right corner.
- $N_{ele} = 5779$, ${}^{h_{min}}/_{h_{max}} = 0.04$
- Injection rate = $5.0 \cdot 10^{-5}\ m^3 s^{-1}$ (CO_2)
- Dimensions: 1.88 x 1.25 m
- With gravity effects.
- $\rho_w = 1045\ kg \cdot m^{-3}$
- $\rho_{nw} = 479\ kg \cdot m^{-3}$

2D Kilve Bed 3 Fractured model (partial)
CPU Run Time Comparison ($k_f / k_m = 4.165 \times 10^6$)



Mindel& Mansipov, SIAM Annual Meeting San Diego, CA, July 2013

Mesh



Implementation for Slightly Compressible two-phase flow

- Assumptions:
  - Flow is slightly compressible, isothermal.
  - Incompressible rock ($\phi = \phi(\mathbf{x})$ and $k = k(\mathbf{x})$).
  - Fluid phases are identical, of constant viscosity, and immiscible.
  - Gravitational and capillary forces are neglected.
  - Both fluids follow the same linear relative permeability model, whereby:

$$k_{r,w} = s_w \qquad \text{and} \qquad k_{r,nw} = 1 - s_w$$

- The mass conservation governing equations can thus be combined to produce,

$$\phi c_f \frac{\partial p}{\partial t} - \nabla \cdot \left(\frac{k}{\mu} \nabla p \right) = 0$$

- The saturation equation, in the absence of external sources and gravitational effects, dropping phase subscripts, may be written as:

$$\phi \frac{\partial s}{\partial t} = -\nabla \cdot (s\mathbf{u}) - \phi s c_f \frac{\partial p}{\partial t}$$

- Darcy velocity, in the absence of gravitational and capillary effects can be written as:

$$\mathbf{u} = -\frac{k}{\mu} \nabla p$$

Implementation for Slightly Compressible two-phase flow

- Using Finite Volume spatial discretization, the discrete form of the flux balance for finite volume cell i for the pressure equation is:

$$\phi_i c_{f_i} \Omega_i \frac{\left(p_i^{n+1} - p_i^n\right)}{\Delta t} = \sum_{j=1}^{n_f} A_j \frac{\bar{K}_j}{\mu_j} (\nabla p)_j^n \cdot \widehat{\mathbf{n}}_\mathbf{j} \qquad (\boldsymbol{Explicit})$$

$$\phi_i c_{f_i} \Omega_i \frac{\left(p_i^{n+1} - p_i^n\right)}{\Delta t} = \sum_{j=1}^{n_f} A_j \frac{\bar{K}_j}{\mu_j} (\nabla p)_j^{n+1} \cdot \widehat{\mathbf{n}}_\mathbf{j} \qquad (\boldsymbol{Implicit})$$

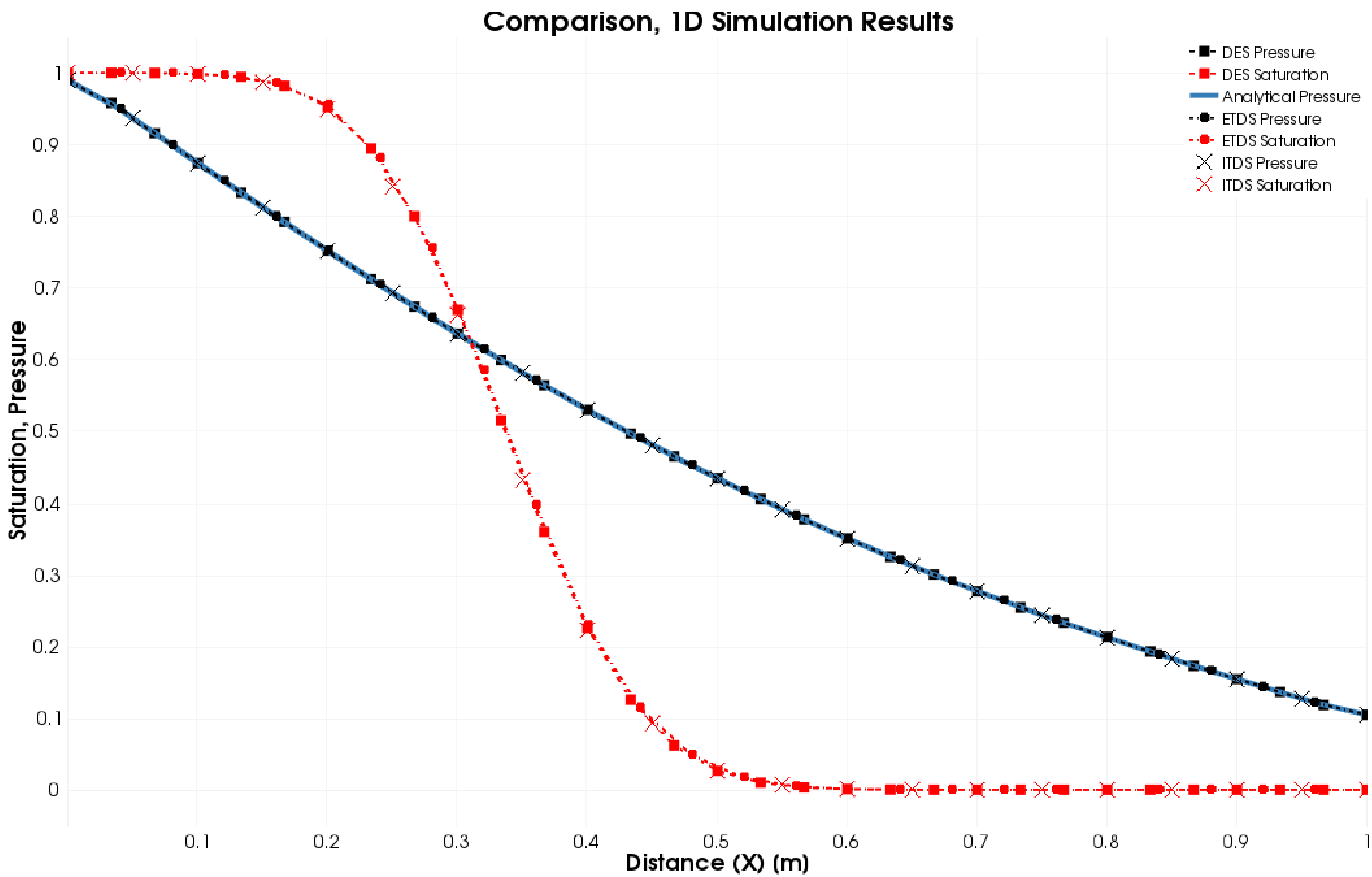
- And the second governing equation is:

$$\phi_i \Omega_i \frac{\left(s_i^{n+1} - s_i^n\right)}{\Delta t} = -\sum_{j=1}^{n_f} A_j s_j^n \mathbf{u}_j^n \cdot \widehat{\mathbf{n}}_\mathbf{j} - \phi_i \Omega_i s_i^n c_f \frac{\left(p_i^{n+1} - p_i^n\right)}{\Delta t} \qquad (\boldsymbol{Explicit})$$

$$\phi_i \Omega_i \frac{\left(s_i^{n+1} - s_i^n\right)}{\Delta t} = -\sum_{j=1}^{n_f} A_j s_j^{n+1} \mathbf{u}_j^{n+1} \cdot \widehat{\mathbf{n}}_\mathbf{j} - \phi_i \Omega_i s_i^{n+1} c_f \frac{\left(p_i^{n+1} - p_i^n\right)}{\Delta t} \qquad (\boldsymbol{Implicit})$$

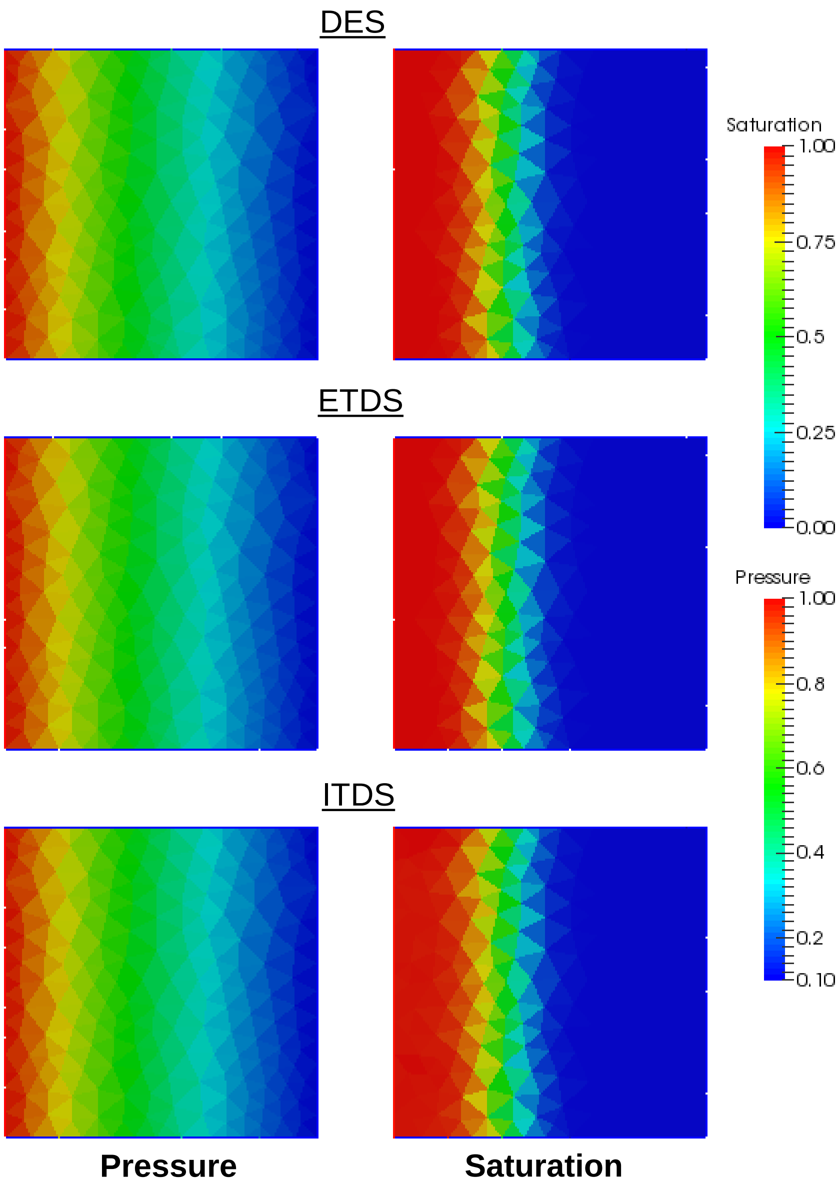
- Where n_f is the number of faces of cell i . A_j is the area of face j. $\widehat{\mathbf{n}}_\mathbf{j}$ is the outward facing unit normal from cell i to cell j.

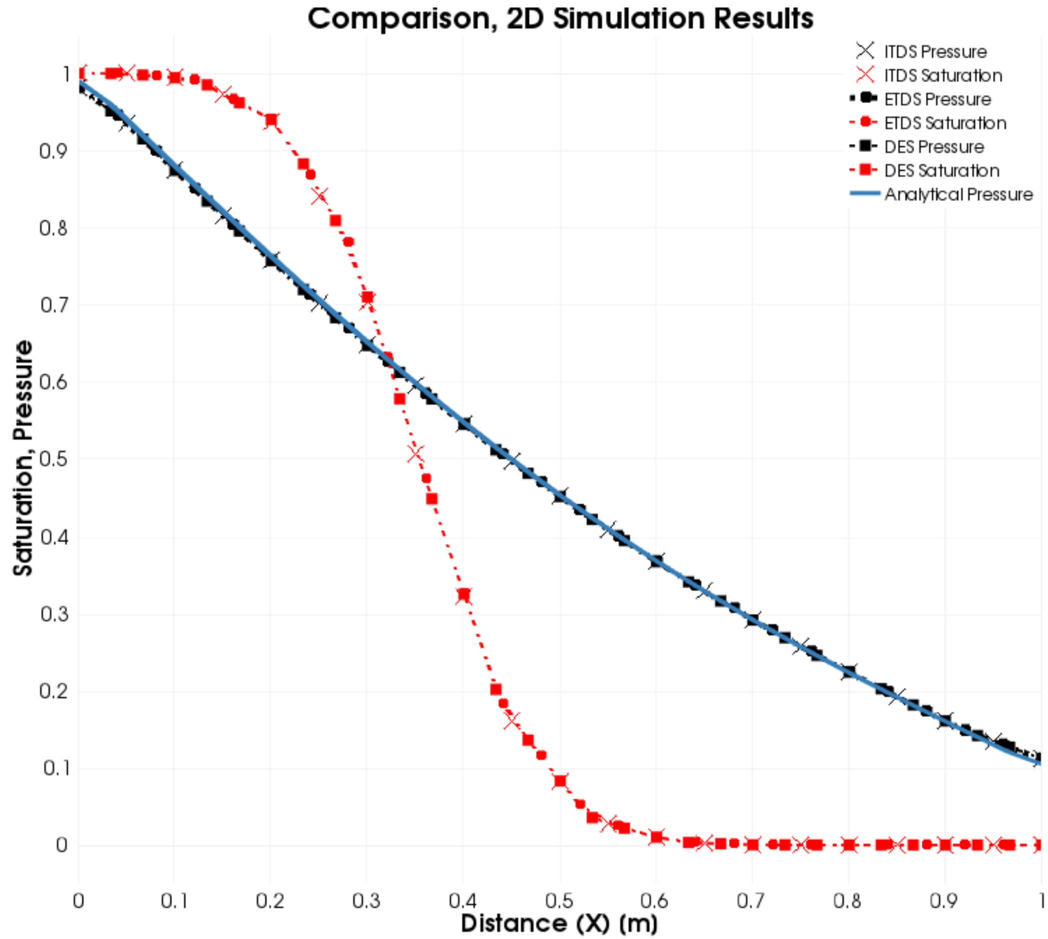
Results



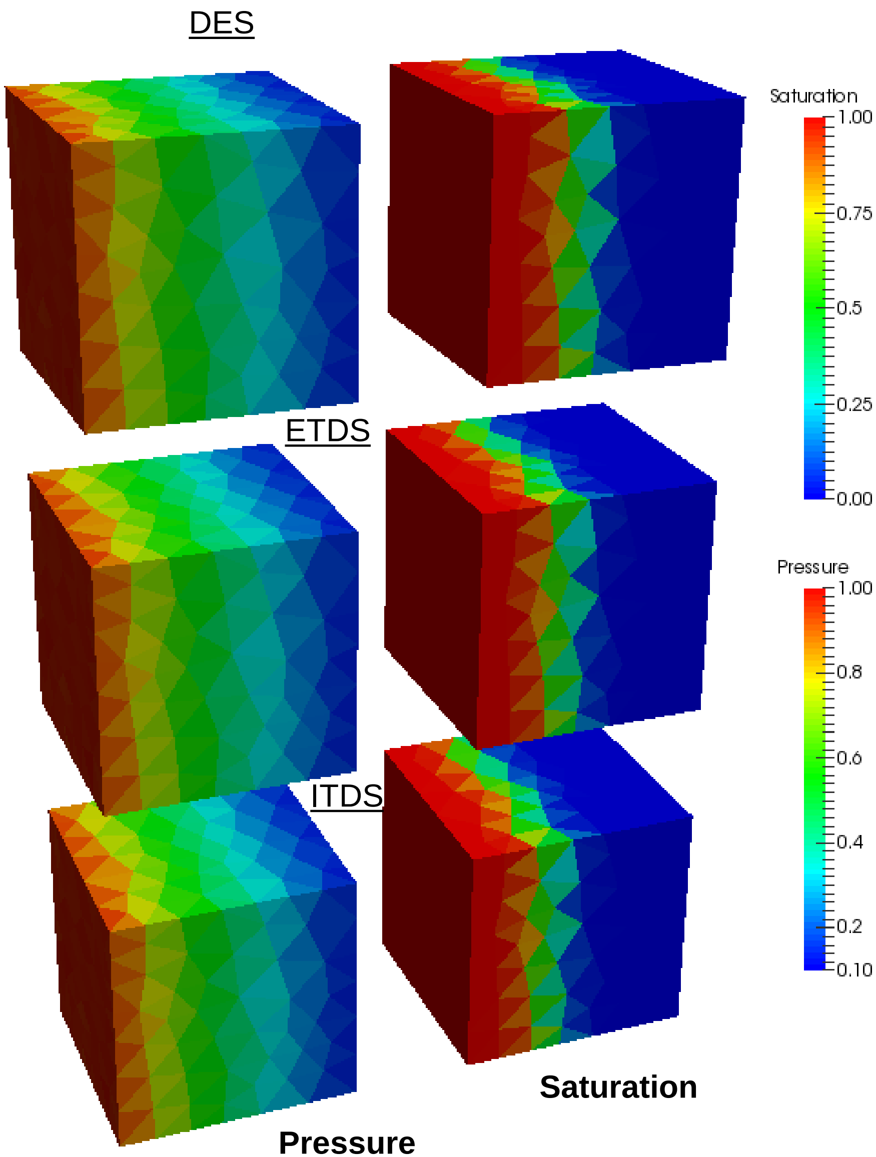
Note: 30 equal sized cells were used

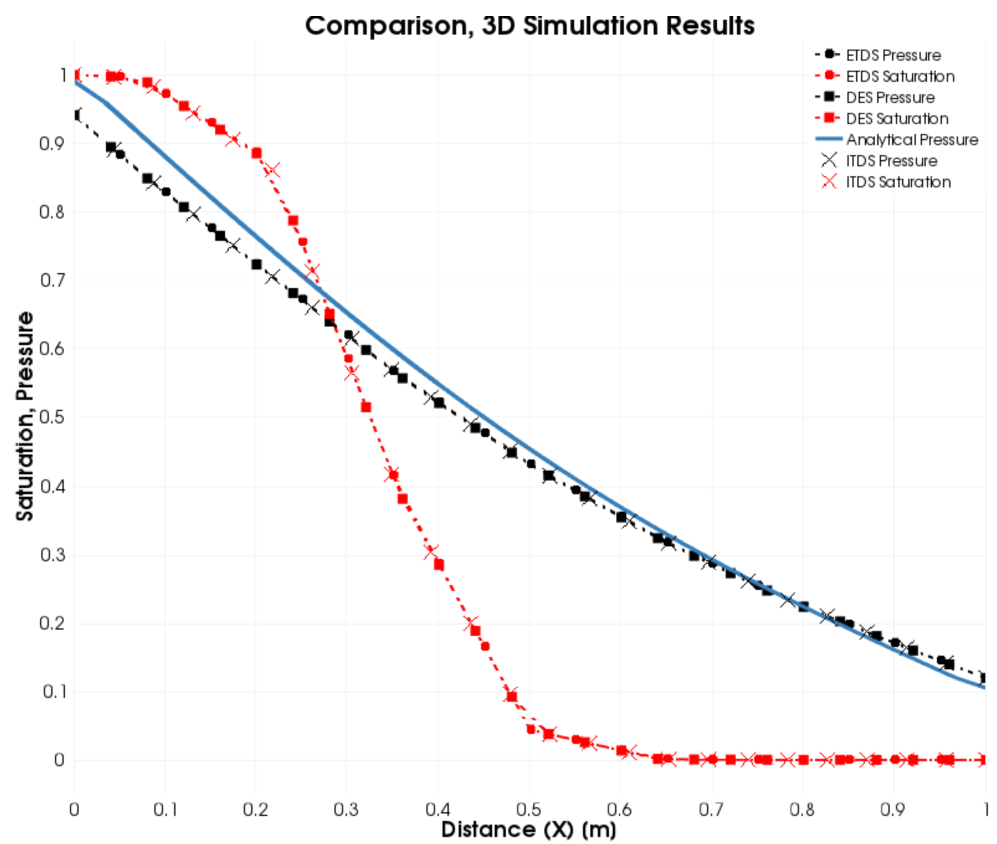
Results


DES
ETDS
ITDS
Pressure
Saturation
Saturation
1.00
0.75
0.5
0.25
0.00
Pressure
1.00
0.8
0.6
0.4
0.2
0.10


Comparison, 2D Simulation Results
ITDS Pressure
ITDS Saturation
ETDS Pressure
ETDS Saturation
DES Pressure
DES Saturation
Analytical Pressure
Saturation, Pressure
Distance (X) (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

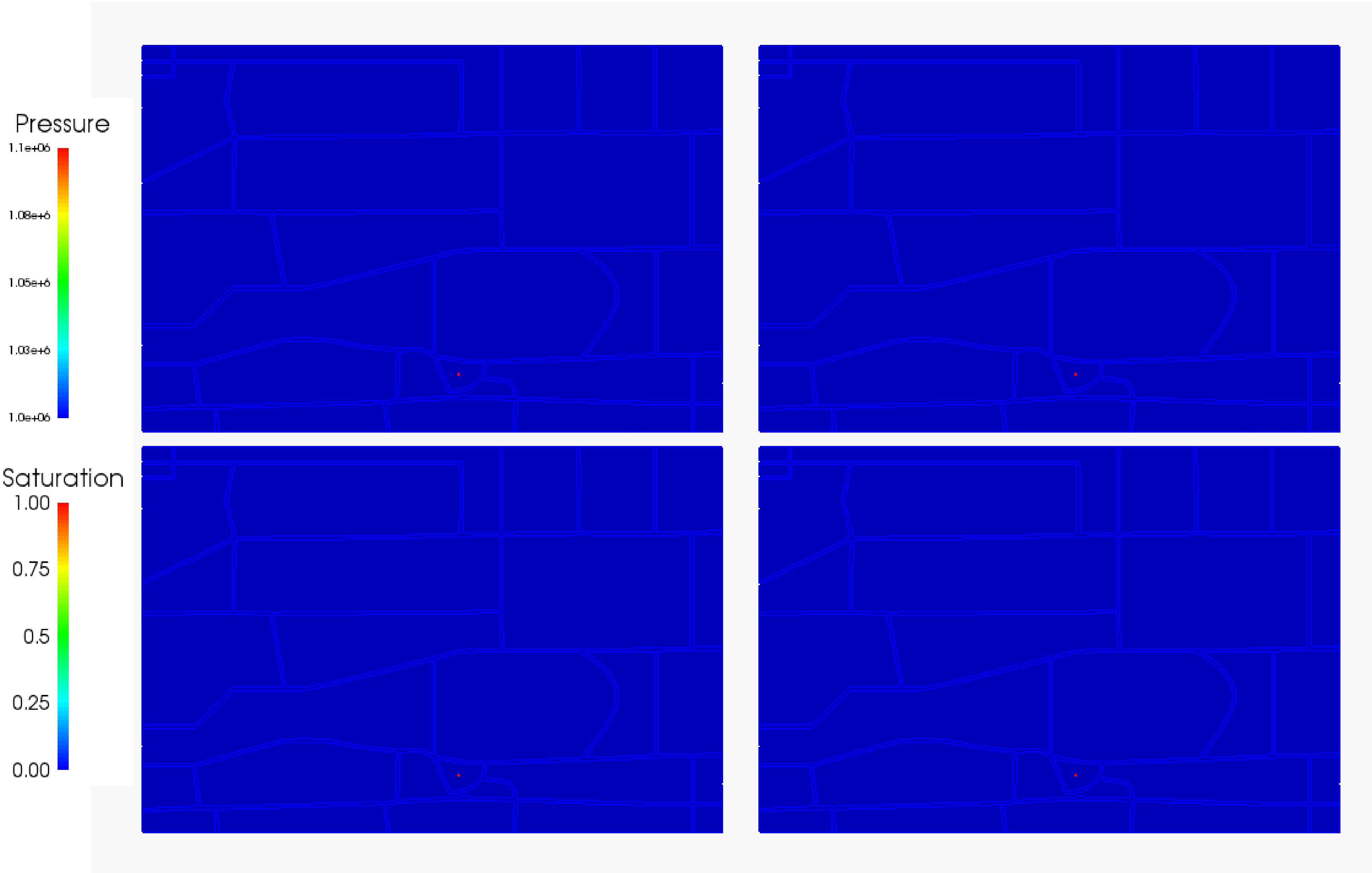
Results


DES
ETDS
ITDS
Saturation
1.00
0.75
0.5
0.25
0.00
Pressure
1.00
0.8
0.6
0.4
0.2
0.10
Saturation
Pressure

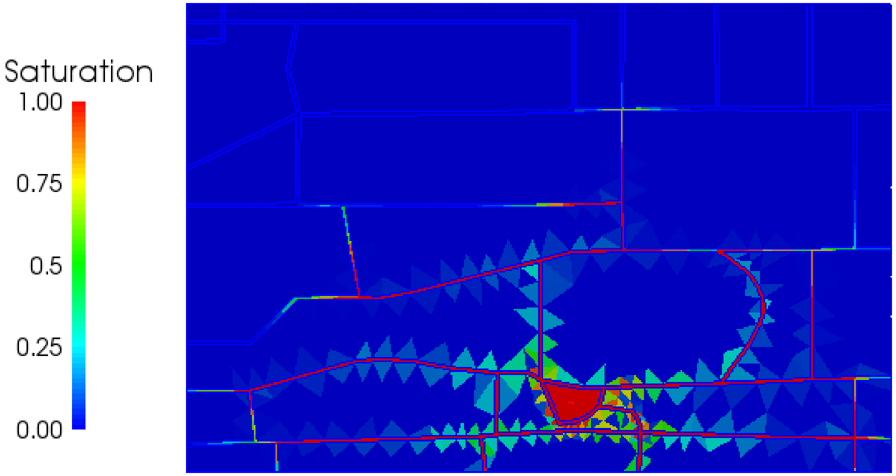

Comparison, 3D Simulation Results
ETDS Pressure
ETDS Saturation
DES Pressure
DES Saturation
Analytical Pressure
ITDS Pressure
ITDS Saturation
Saturation, Pressure
Distance (X) (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

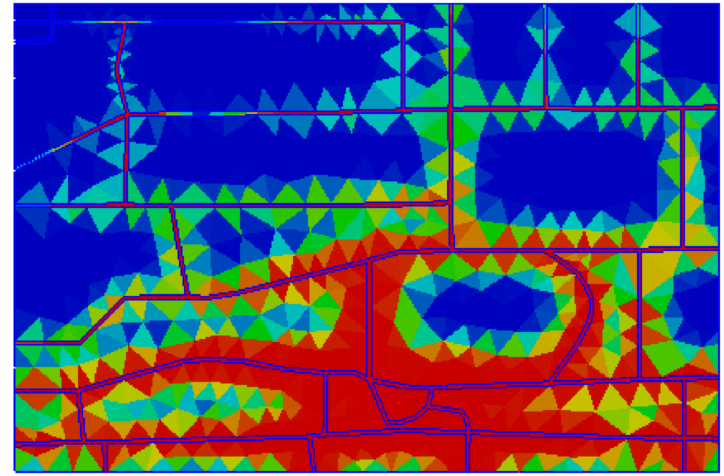
Results

K_f/K_m=10000 K_f/K_m=1000

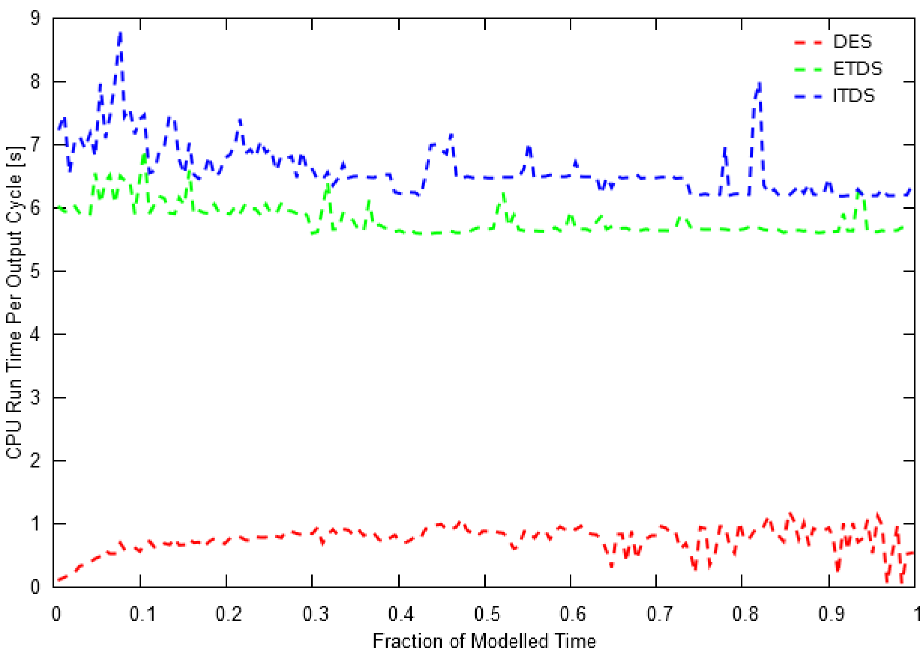


Results

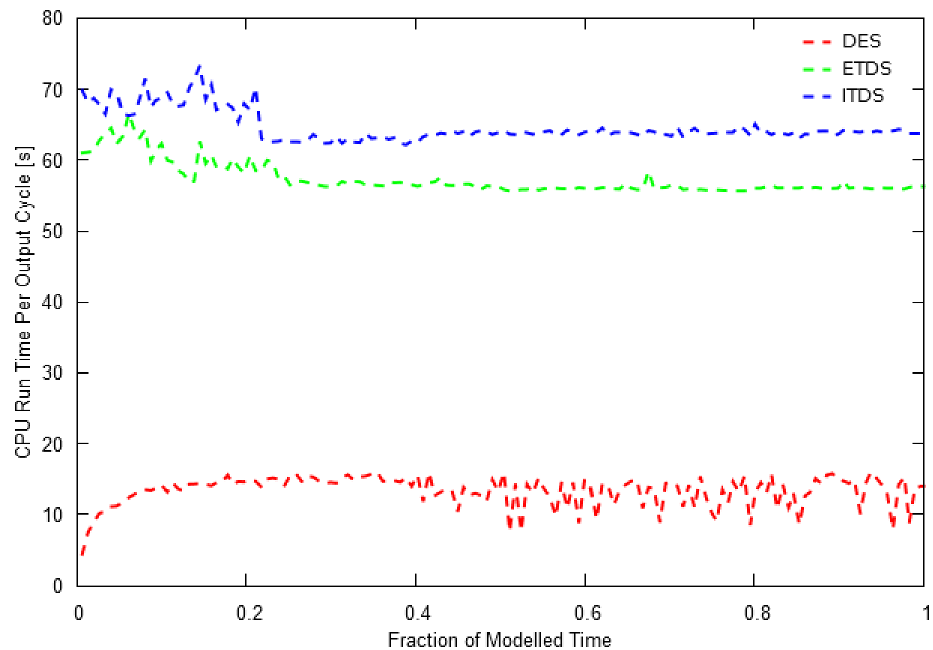




2D Kilve Model - CPU Run Time per Cycle Comparison (k_f / k_m=10000, cycle =0.05 [s])

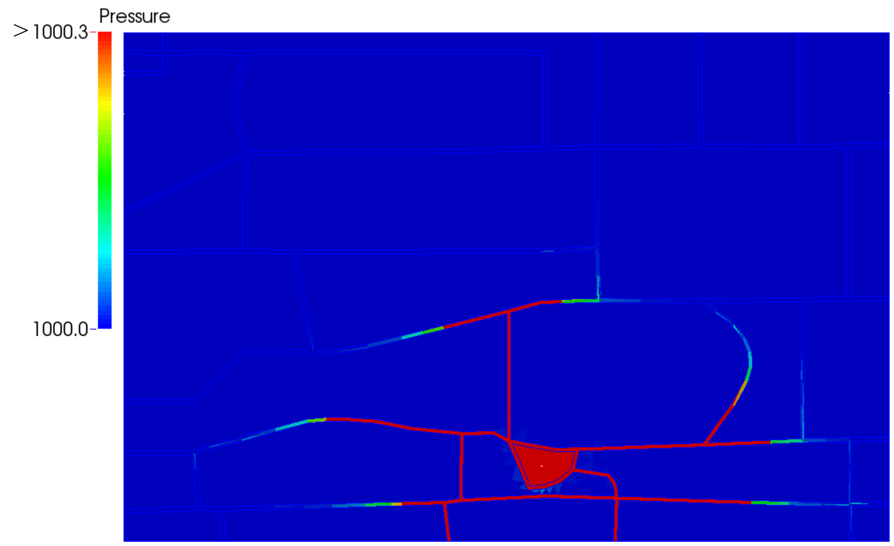


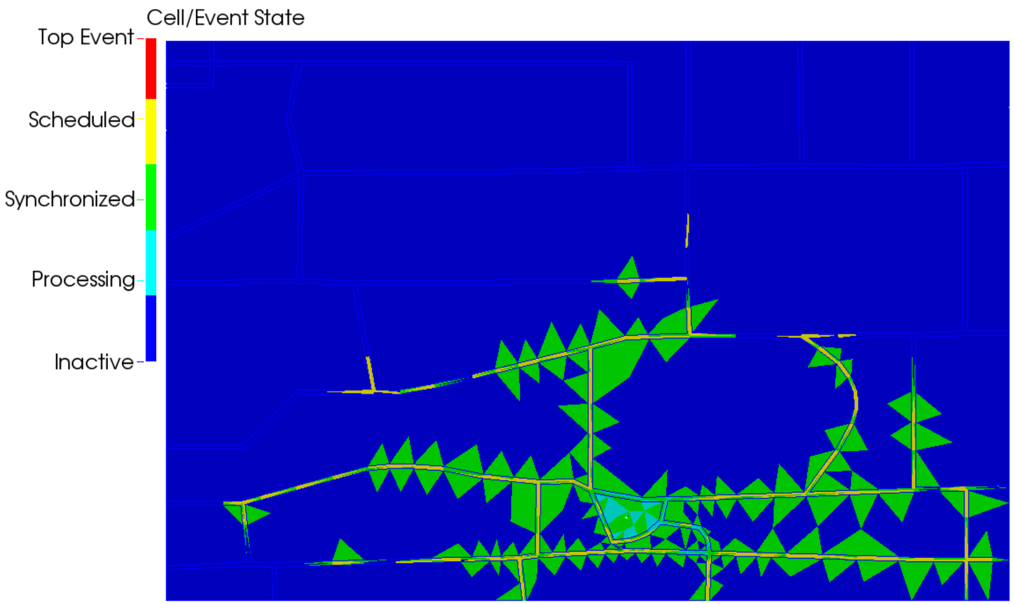
2D Kilve Model - CPU Run Time per Cycle Comparison (k_f / k_m=1000, cycle =0.05 [s])

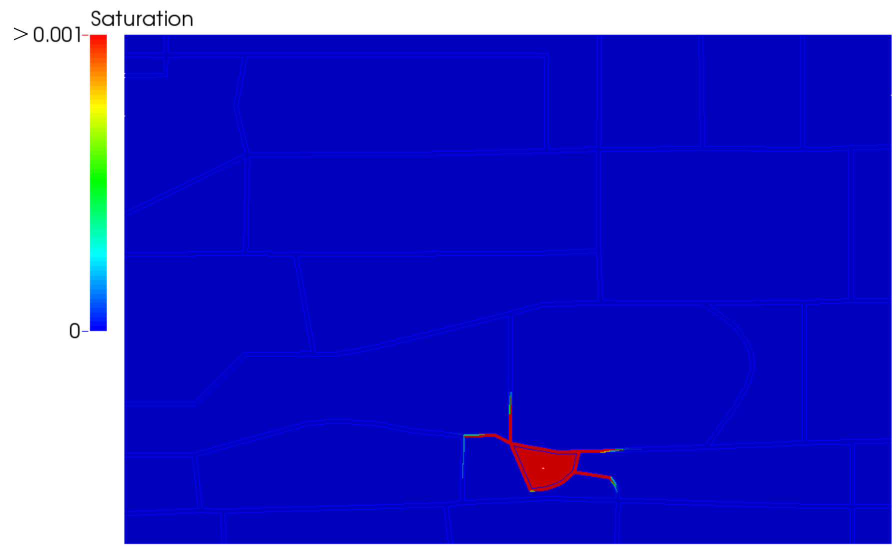


Results

- Snapshot showing instantaneous operation activity within a DES step.
- Variable range of the frames on the right has been cropped to show how far Pressure and Saturation propagate, activating cells in their path







Summary and work in progress

- An asynchronous, event-based methodology has been applied to slightly compressible two-phase fluid flow in heterogeneous porous media.
- Demonstrated good performance vs. ETDS and ITDS schemes for scenarios of high material heterogeneity, non-linearity, and resolution variation.
- Our work in progress includes (…but is not limited to…),
  - **Efficiency:** work on better calculation of target variable changes.
  - **Parallelism**
  - **Multiple variables** from more complex systems of PDE's, with application to, for example, reactive transport modelling, etc..
  - **Smart algorithm:** Larger number of applied cases need to be studied to understand when the method might not be a viable option, thus automatically switch to TDS schemes or other approaches. (e.g. when large portions of the domain become active)

References

Courant, R., Friedrichs, K. and Lewy, H. (1928). Ueber die partiellen differenzengleichungen der mathematischen physik, Mathematische Annalen 100: 32–74.

Karimabadi H. , Driscoll J., Omelchenko Y.A., Omidi, N. (2005), A new asynchronous methodology for modelling of physical systems: breaking the curse of the Courant condition, Journal of Computational Physics, Volume 206, Issue 2, Pages 755-775

Lew A. , Marsden J. E. , Ortiz M. , and West M. . Asynchronous variational integrators. *Archive for Rational Mechanics and Analysis* 167(2), 85-146, 2003.

Nutaro J., Zeigler B. P., Jammalamadaka R., and Akerkar S.. (2003). Discrete event solution of gas dynamics within the DEVS framework. In *Proceedings of the 2003 international conference on Computational science* (ICCS'03),Springer-Verlag, Berlin, Heidelberg, 319-328.

Omelchenko Y.A., Karimabadi H. (2006), Self-adaptive time integration of flux-conservative equations with sources, Journal of Computational Physics, Volume 216, Issue 1, Pages 179-194, ISSN 0021-9991