MODELING OF FLOW TRANSPORT IN POROUS MEDIA, FROM PORE SCALE TO NON-DARCY FLOW

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Outlines

Introduction

- CO_2 sequestration & Porous media
- Imaging and tomography
- Flow modelling

Flow in carbonate rocks

- Homogeneity
- Convection term effect
- Flow direction

Non-Darcy flow in Carbonates

Future works & challenges

Conclusions

Notations

[A] square matrix of coefficients

d Vector between two cells centers

K Vector in the non-orthogonality treatment

S Source term

S surface area vector

t time

V Volume

Δ Difference

Γ Diffusivity

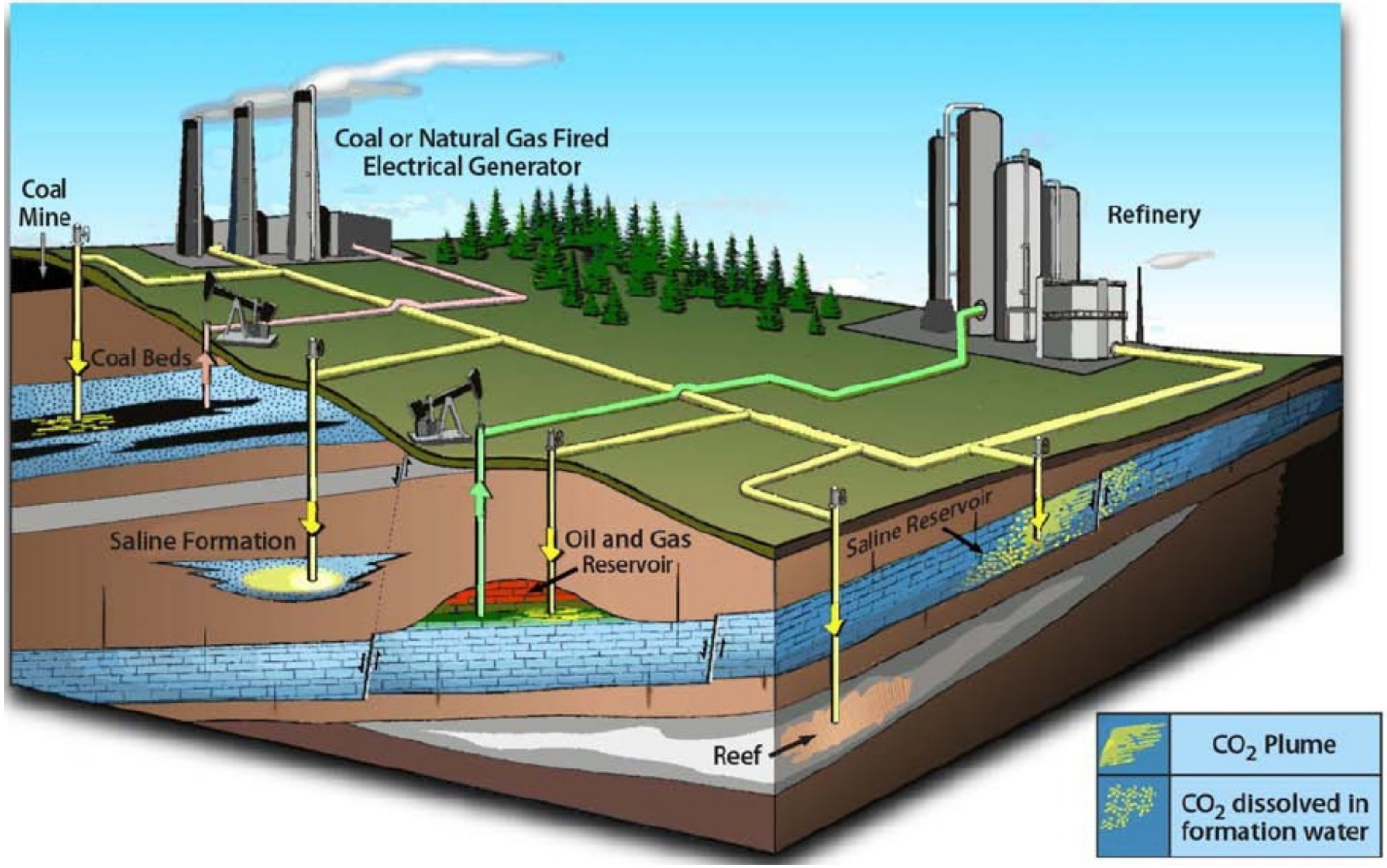
λ Under-relaxation factor

μ Dynamic viscosity

ρ Density

Φ tensorial quantity

Geological Carbon Storage (GCS)

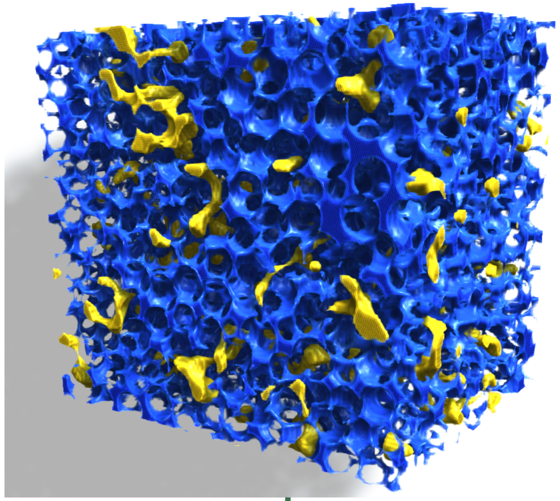


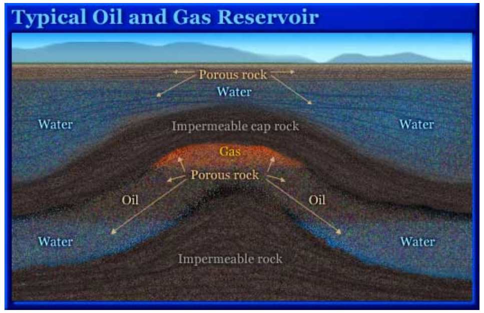
(Adapted from original figure, courtesy of Dan Magee, Alberta Energy Utilities Board, Alberta Geologic Survey, 2008.)

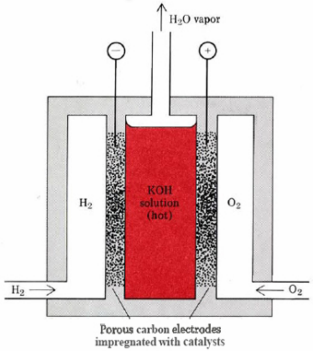
Porous Media

Materials which contain pores and skeletal

✓Pores: fluid (gas and liquid)
✓Skeletal: solid







Scientific Questions

How to get more insight into fluid flow in porous media?

How can we capture the complexity of porous media (specially carbonate rocks)?

Different effects in pore scale modelling?

How does system chenge when velocity increases?

Imaging

Tomography:

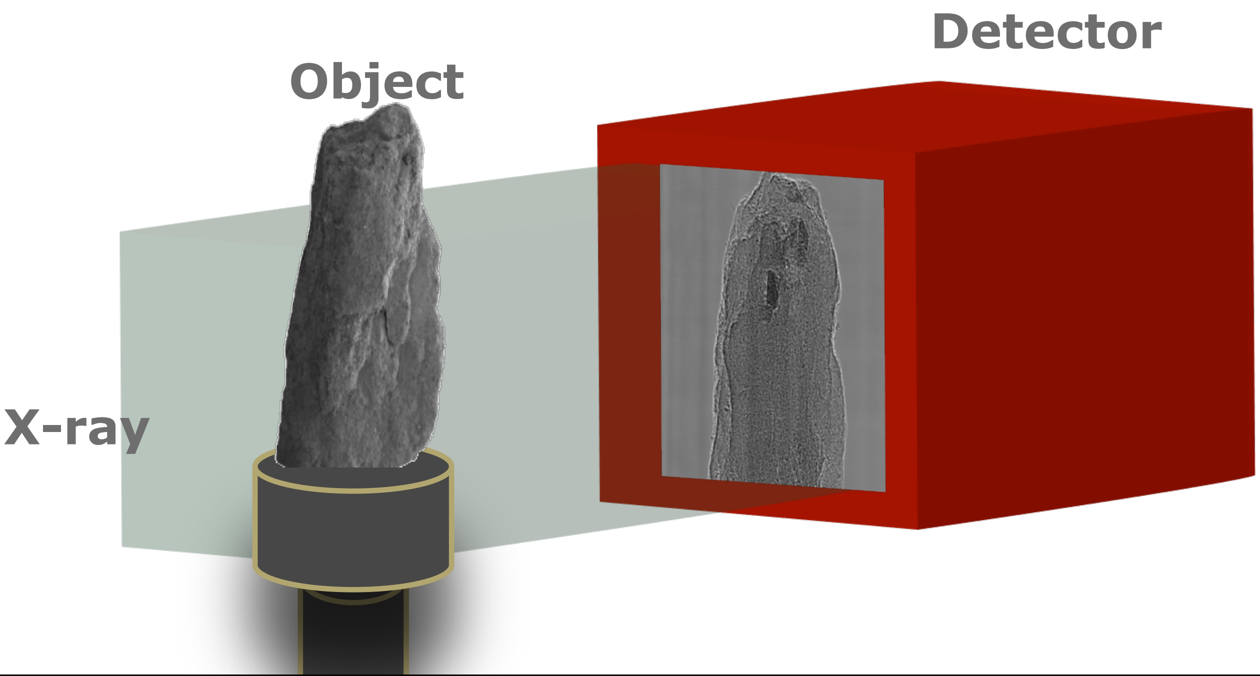
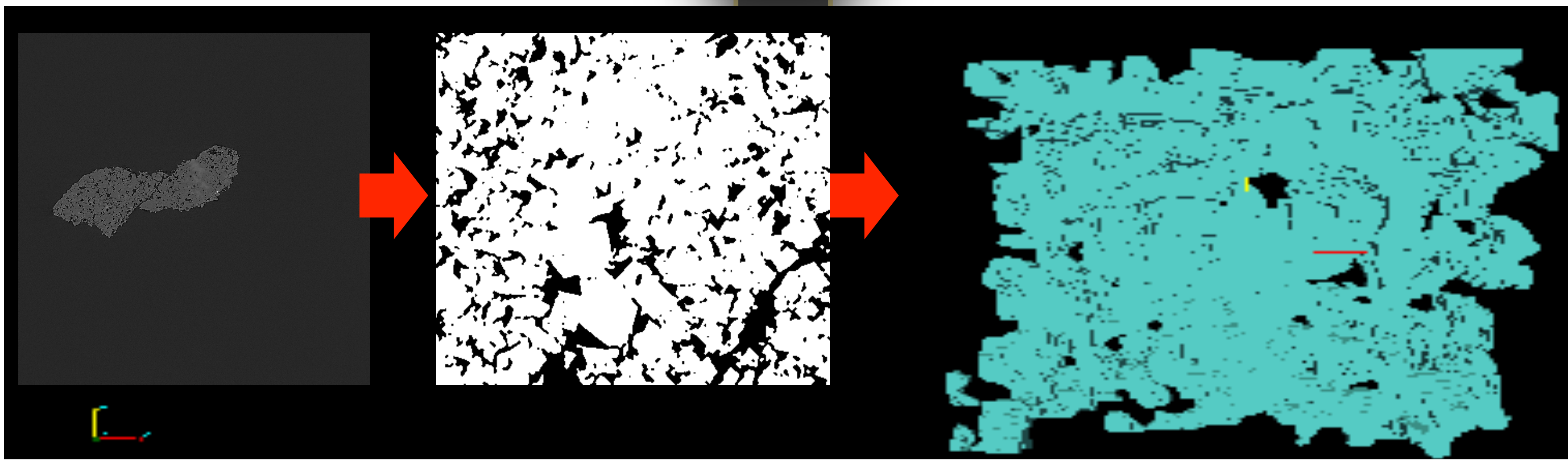


Image segmentation:



Pore Scale modelling Methods

Lattice Boltzman Method (LBM)

Pore Network Model

Navier-Stokes equations

Finite difference

Finite element

Finite Volume

Advantages - Disadvantages

LBM

Particle distribution functic

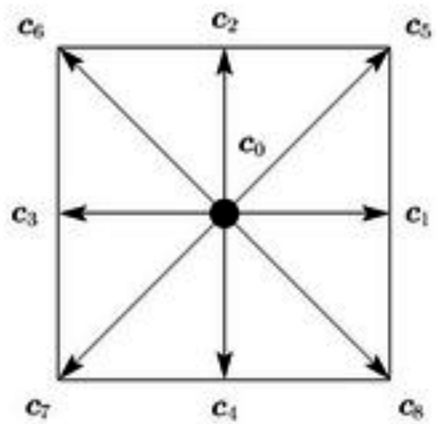
$$f_i(x + v_i, t + 1) - f_i(x, t) = \Omega_i(f_i(x, t))$$

Collision:

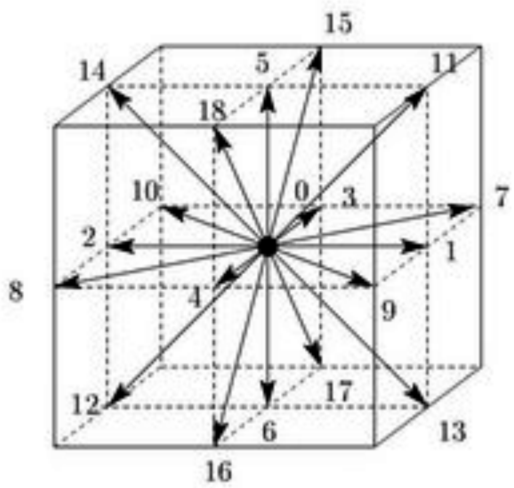
$$\Omega_i = -\frac{1}{\tau}(f_i - f_i^{eq}) \qquad \sum_i \Omega_i = 0 \quad and \quad \sum_i \Omega_i v_i = 0$$

Momentum and densitv:

$$\sum_i f_i = \rho \quad and \quad \sum_i f_i v_i = \rho u$$



D2Q9



D3Q19

Navier-Stokes

Momentum equation

$$\rho\left(\frac{\partial v}{\partial t} + v\,.\nabla v\right) = -\nabla p + \mu\,\nabla^2 v$$

Continuity equation

$$\nabla.v = 0$$

Expanding equations in one-direction:

$$\rho\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\,\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Equation Discretisation

Finite Volume Method

$$\underbrace{\frac{\partial \rho\phi}{\partial t}}_{\text{time derivative}} + \underbrace{\nabla \bullet (\rho \mathbf{U} \phi)}_{\text{convection term}} = \underbrace{\nabla \bullet (\Gamma \nabla \phi)}_{\text{diffusion term}} + \underbrace{S_\phi(\phi)}_{\text{source term}}$$

Gauss theorem:

$$\int_V \nabla \otimes \phi \, dV = \int_S d\mathbf{S} \otimes \phi$$

Convection:

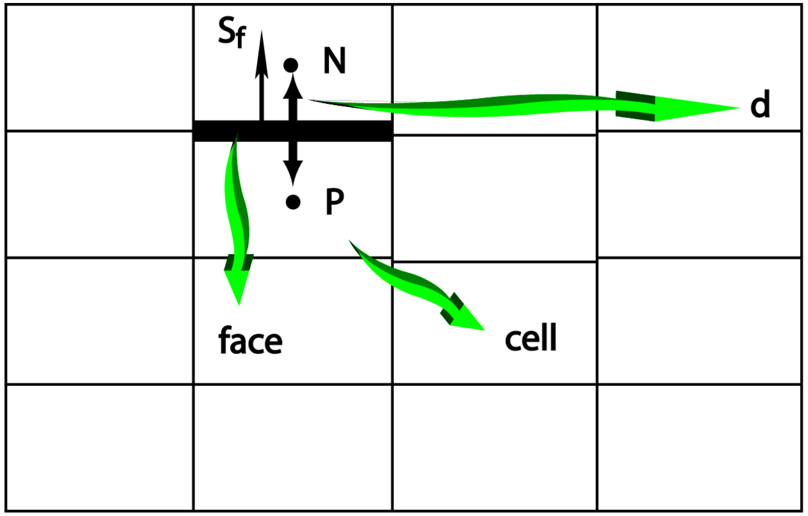
$$\int_V \nabla \cdot (\rho U \phi) dV = \int_S dS \cdot (\rho U \phi) = \sum_f S_f \cdot (\rho U)_f \phi_f = \sum_f F \phi_f$$

Laplace:

$$\int_V \nabla \cdot (\Gamma \nabla \phi) dV = \int_S dS \cdot (\Gamma \nabla \phi) = \sum_f \Gamma_f S_f \cdot (\nabla \phi)_f$$

Divergence:

$$\int_V \nabla \cdot \phi \, dV = \int_S dS \cdot \phi = \sum_f S_f \cdot \phi_f$$



Equation Discretisation

Diffusion:

$$\int_V \nabla \bullet (\Gamma \nabla \phi)\, dV = \int_S d\mathbf{S} \bullet (\Gamma \nabla \phi) \approx \sum_f \Gamma_f \left(\mathbf{S} \bullet \nabla_f \phi\right)$$

For non-orthogonal meshes, correction term as below is used to preserve the second order accuracy

$$\mathbf{S} \bullet \nabla_f \phi = \underbrace{|\mathbf{\Delta}| \nabla_f^{\perp} \phi}_{\text{orthogonal contribution}} + \underbrace{\mathbf{k} \bullet (\nabla \phi)_f}_{\text{non-orthogonal correction}}$$

Where Δ and k are vectors which will be calculated by non-orthoganality treatment

Source Term:

And is linearized as

$$S_\phi(\phi) = \phi S_I + S_E$$

Where S_E and S_f can be dependence on Φ. The source term is integrated over control volume as

$$\int_V S_\phi(\phi) dV = S_I V_P \phi_P + S_E V_P$$

Solution

SIMPLE method:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial p}{\partial x_i}\right) = -\frac{\partial}{\partial x_i}\left[\frac{\partial(\rho u_i u_j)}{\partial x_j}\right]$$

Relaxation factor:

$$u^{n+1} = u^* + \alpha_u u'$$

Co. Number:

$$Co = \frac{U_f . d}{|d|^2 \Delta t}$$

Boundary condition:

Fixed value:

$$S_f . (\nabla\phi)_f = |S_f| \frac{\phi_b - \phi_P}{|d|}$$

In this way, it is second order accurate if Φ_b is constant and otherwise it's first order accurate

Fixed gradient:

$$g_b = (\frac{\boldsymbol{S}}{|S|} \cdot \nabla\phi)_f$$

Solution Technique for linear Algebraic Equations

generic form of linear algebraic equations:

$$a_P\phi_P^n + \sum_N a_N\phi_N^n = R_P$$

Where Φ^n_p depends on the neighbouring cells

The system of algebric equations can be expressed in a matrix form of

$$[A][\phi] = [R]$$

[A] ia a sparse square matrix with coefficient a_p on the diagonal and a_N off the diagonal. [Φ] is dependent variable and [R] is the source vector.

The matrix [A] can be decomposed into two matrices, the diagonal [D] and off-diagonal [N], such as:

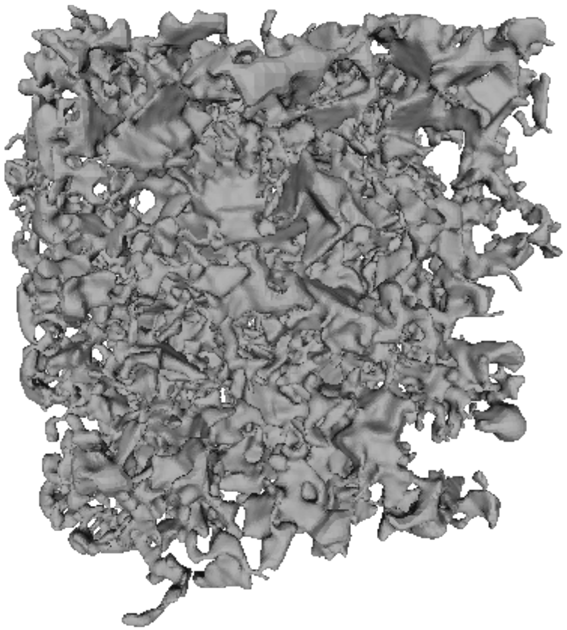
$$[A] = [D] + [N]$$

Conjugate Gradient method has been used or solution

K-C method

Permeability

$$k = C\,\frac{\varphi^3}{S_V^2} = C\,\frac{\varphi^3}{(A_{BET}\rho_c)^2}$$



A_{BET} is the Brunaver Emmett Teller surface area in m^2/g

$$A_{BET} = \frac{\sum S_n}{\rho_c N_c l_{vox}}$$

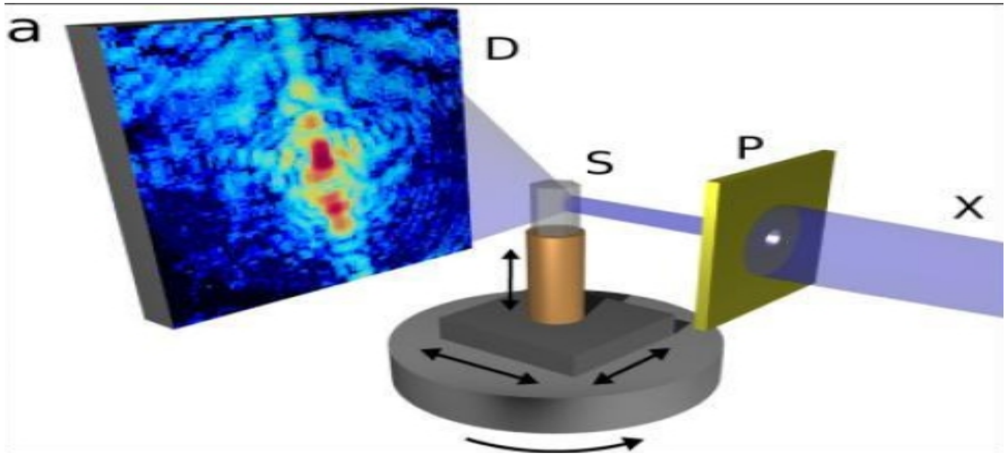
porosity of the material

$$\varphi = 1 - \frac{\langle \mu_i \rangle}{\mu_c}$$

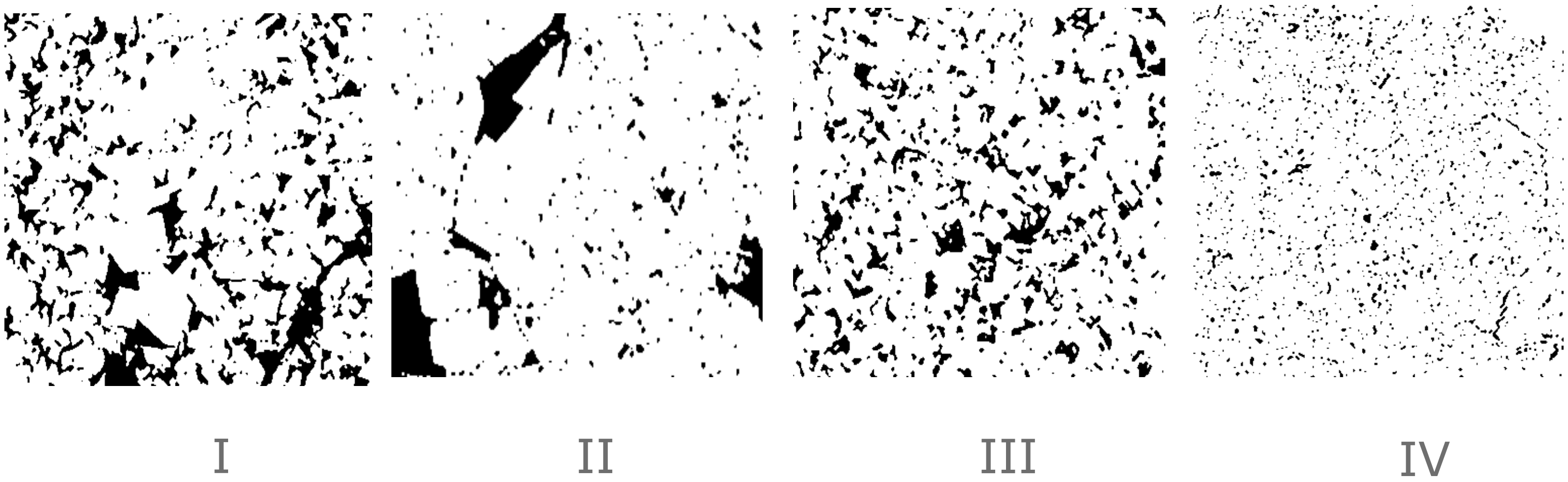
Parameters

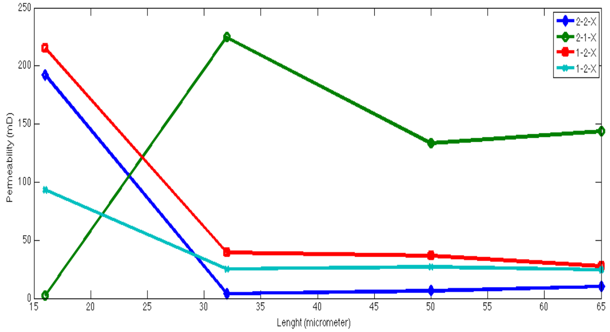
- ✓High-resolution image
- ✓Homogeneity
- ✓Equation effect
- ✓Flow direction

High Resolution Images



Dierolf et al. (2010)





Change of permeability along samples

Sample Volume	Porosity	Kozeny-Carman(mD)	This work(mD)
1-1-X	17.10	20.4	24.6135
1-2-X	15.66		28.1885
2-1-X	16.62	12.6	143.6578
2-2-X	16.97		10.1517

Comparison with different methods

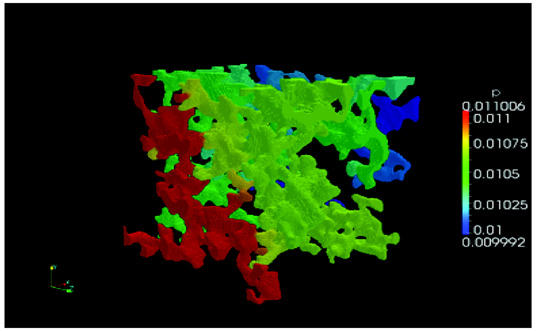
Different Effects

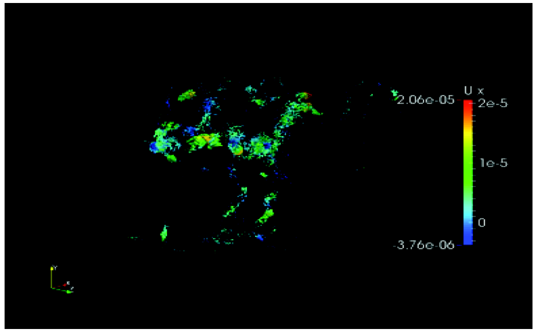
Sample Name	Per. y (mD)	Per. z (mD)	Change y (%)	Change z (%)
1-1	33.34	23.21	35	5
1-2	10.35	24.87	63	11
2-1	3.19	17.61	97	87
2-2	11.25	8.55	10	15

Different direction effect

Sample Name	Permeability (without convective force)(mD)	Difference
1-1-X	24.5992	5.0e-4
1-2-X	28.1870	5.3e-5
2-1-X	143.6584	4.5e-6
2-2-X	10.1523	6.4e-5

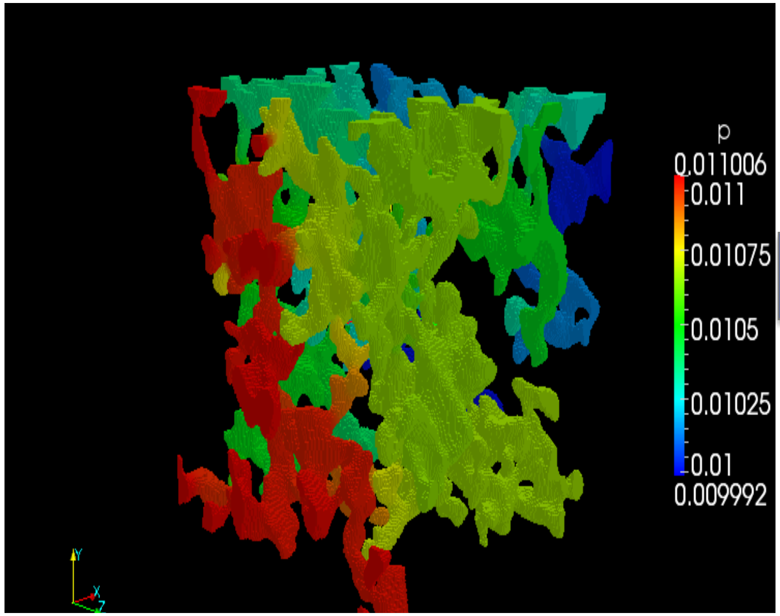
Convective term effect

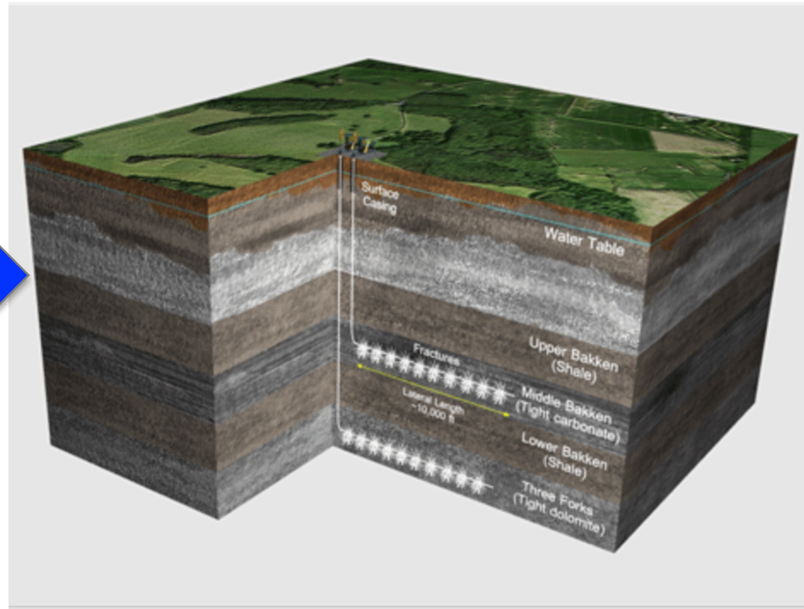




Pressure and Velocity field

Non-Darcy

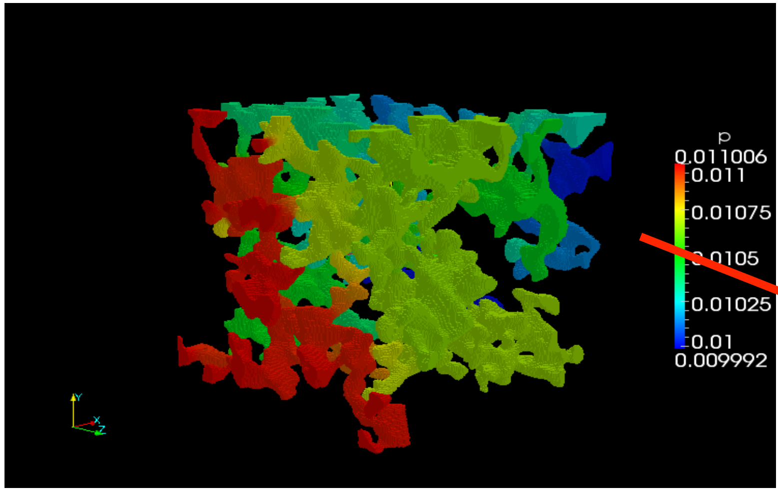




Extracting intrinsic permeability and non-Darcy flow parameters

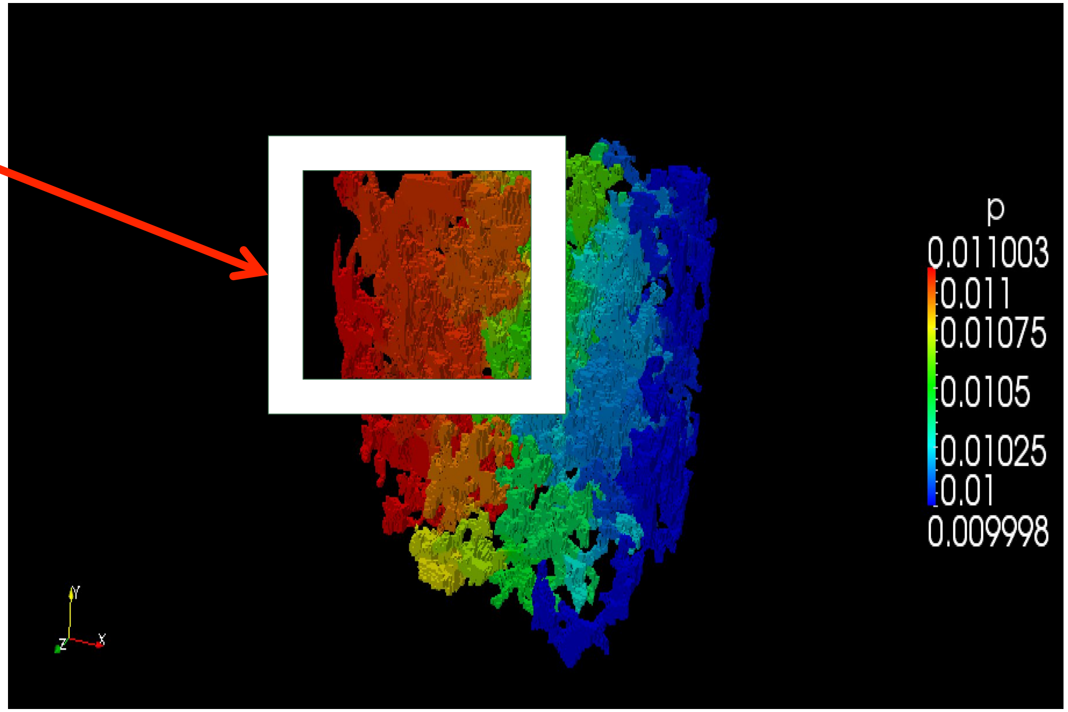
Non-Darcy

Representative volume:



100^3 voxel

8 times bigger volume



200^3 voxel

Non-Darcy

Darcy-Forchheimer: $$-\frac{\partial \mathrm{p}}{\partial \mathrm{X}} = \frac{\mu \mathrm{v}}{\mathrm{k}} + \mu\beta \mathrm{v}^2$$

Darcy-Brinkmann: $$-\frac{\partial \mathrm{p}}{\partial \mathrm{X}} = \frac{\mu \mathrm{v}}{\mathrm{k}} - \mu\nabla^2 \mathrm{v}$$

Momentum: $$\frac{\partial}{\partial t}(\gamma\rho u_i) + u_j\frac{\partial}{\partial x_j}(\rho u_i) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial \tau_{ij}}{\partial x_j} + S_i$$

Implicit method:

$$\frac{\partial}{\partial t}\int_V \rho\phi\, dV = \frac{(\rho_p\phi_p V)^n - (\rho_p\phi_p V)^0}{\Delta t}$$

$$\phi^n = \phi(t+\Delta t)$$

$$\phi^0 = \phi(t)$$

Where Φ^n is the new value at the time step we are solving and Φ^0 is the old value

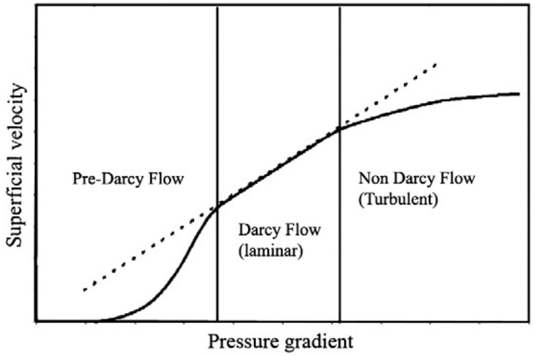
This method is first order accurate in time and guarantees the boundedness of the solution and is unconditionally stable

Non-Darcy

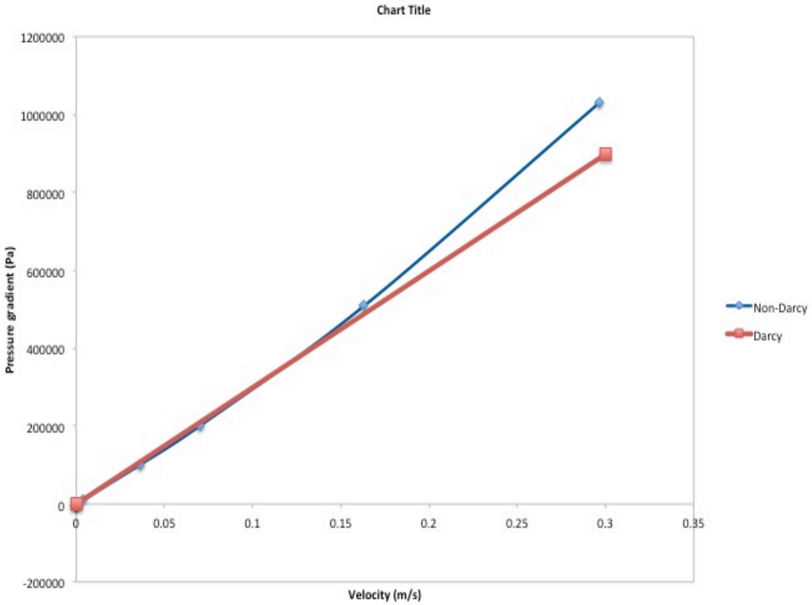
Beta coefficient:

Model	equation	beta
Pascal et al.	$4.8{\times}10^{12}{\times}k^{-1.176}$	1343837
Coles and Hartman	$1.07{\times}10^{12}{\times}\varphi^{0.449}{\times}k^{-1.88}$	47,77
Janicek and Katz	$1.82{\times}10^{8}{\times}\varphi^{0.75}{\times}k^{-1.25}$	7615,90
This work	Direct Modelling	2694,2

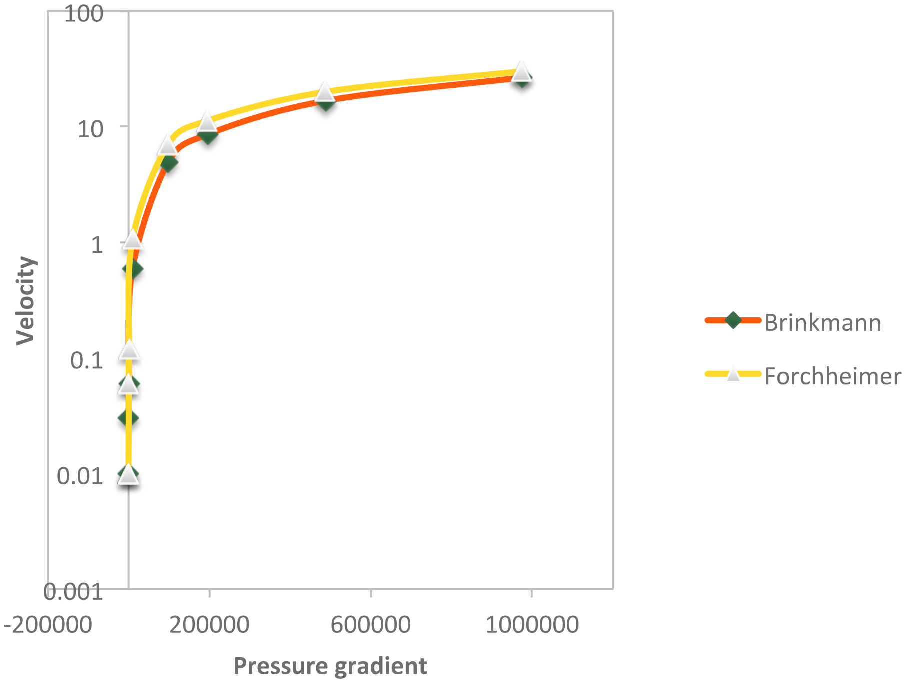
Non-Darcy



Schematic figure



Our results



Comparison of different models

Problems and Future Work

REV: Representative elementary volume

Heterogeneity

Weak verification of macro results by micro results

Morphology change

Conclusions

- Small length scale can cause high instability (high Co. number) in Navier-Stokes flow

- Flow direction has a significant effect on results for complicated porous media (carbonate rocks)

- K-C method can be applied in homogenous porous media

Acknowledgment

Maersk Oil/Gas A/S for P^3 project

the European Community for the CO2-REACT project

CINEMA: "The Alliance for imaging of Energy Materials"

The End

Thank you for your attention