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An up-wind finite volume method on non-orthogonal quadrilateral meshes for the convection diffusion equation in a porous media

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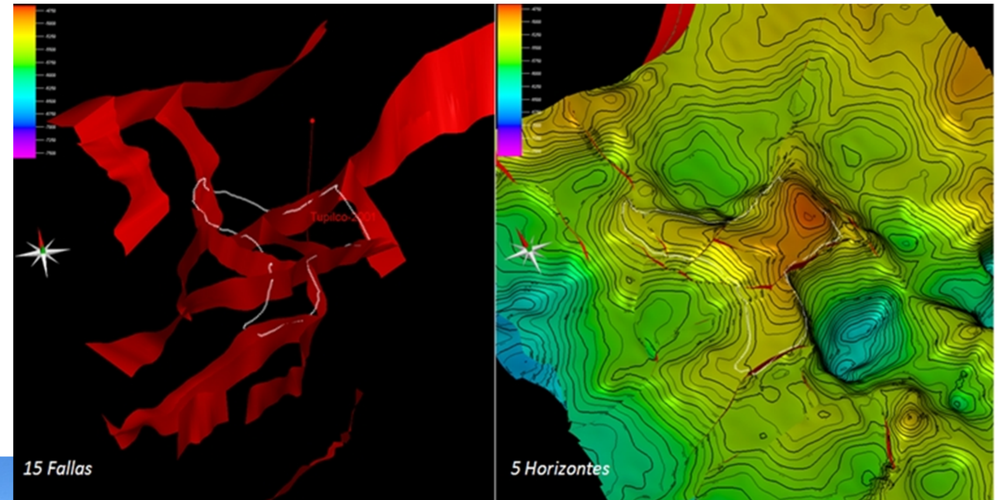
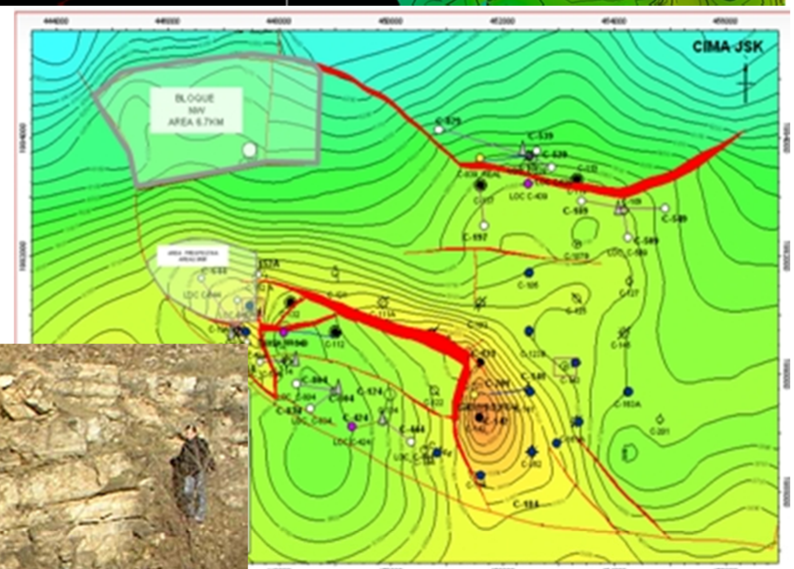
Motivation

Complexity of the reservoir structure (preferential flow channels, faults, areas of high permeability contrast, changes in sediment type, etc.).





Abstract

Traditionally, the finite difference method has been applied to solve numerically the PDE governing fluid flow on a rectangular mesh in porous media for its easy implementation and computational efficiency.

The aim of this work is to solve the two-dimensional convection diffusion equation on a non-rectangular grids formed only by quadrilaterals honouring the internal structures of a reservoir (preferential flow channels, faults, areas of high permeability contrast, changes in sediment type, etc.), taking into account different physical configurations of the porous medium.

To take advantage of the good representation of the domain through these meshes, the finite volume method was used, which is conservative and facilitates the treatment of the boundary conditions. In this method the convection diffusion equation is integrated on each quadrilateral (control volume) of the mesh, thus obtaining the integral form of the equation. The velocity value in the face of each quadrilateral is determined according to the direction of the flow (upwind scheme). After approximating the integrals involved and taking into account the boundary conditions, a discrete equation in each control volume showed up. Finally, a large sparse linear system is obtained, generally non-symmetric and ill-conditioned, which can be solved by iterative methods such as GMRES with incomplete LU preconditioning.

Different scenarios were considered varying boundary conditions (Dirichlet and Neumann type), source term, and diffusion constant fluid velocity. The results are consistent with the physical interpretation of each configuration.

Outline

1. Formulation of the problem
2. Structured quadrilateral meshes
3. Numerical model
   - Steady state
   - Boundary conditions
   - Transient state
   - Structure of the lineal system
4. Results
5. Conclusions

Formulation of the problem

To solve the convection-diffusion equation

$$\frac{\partial \phi}{\partial t} = \nabla \cdot \left(k \nabla \phi(\vec{r}, t)\right) - \vec{v} \cdot \nabla \phi + f$$

in 2D on an structured quadrilateral mesh using the finite volume method considering

- a rectangular domain $\Omega \subset R^2$
- Dirichlet or Newman type conditions on the each domain edge
- the velocity $\vec{v}$ small enough

Structured quadrilateral meshes

Given a two-dimensional domain whose boundary is a closed polygonal line with internal boundaries defined also by polygonal lines, it is required to generate a grid consisting only by quadrilaterals with the following features:

- be conformed, that is, to be a partition of the two-dimensional domain such that the intersection of any two quadrilaterals is a vertex, an edge or empty (never a portion of one edge)
- be structured, which means that only four quadrilaterals meet at a single node and the quadrilaterals that make up the grid need not be rectangular, and
- the mesh generated must rely on the internal boundaries.

It is also considered the possibility that interior points of the domain to be vertices of the resulting grid. These points will represent wells on the reservoir to simulate.

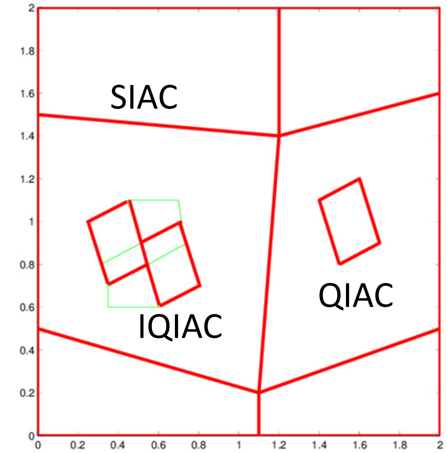
Internal boundaries, that may have a complex layout and configuration, mimic the structure of the reservoir.

Structured quadrilateral meshes

The fundamental technique for generating such grids, is the deformation of an initial Cartesian grid and the subsequent alignment with the internal boundaries.

The internal boundaries will be modeled using four types of polygonal lines:

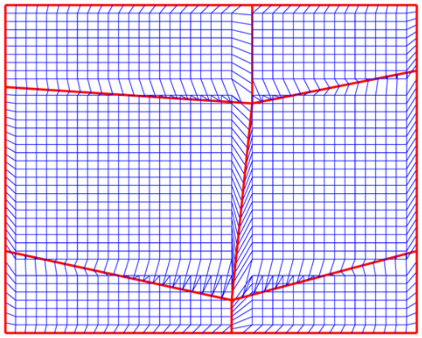
- An open polygonal line strictly included in the domain, from now on an IAC (Internal Alignment Curves).
- An open polygonal line extended to the boundaries of the domain, from now on a SIAC (Spanned IAC).
- A convex four side polygon, denoted as QIAC (Quadrilateral IAC).
- An intersected set of QIAC, denoted as IQIAC.



Structured quadrilateral meshes

The proposed general process for generating the desired mesh, follows these steps:

- Decompose the QIAC and IQIAC into a set of SIAC
- Extend the IAC to the external boundaries to form SIAC
- Generate the initial uniform Cartesian grid
- For each SIAC (including the original SIAC)
    - ✓ Associate a line from the initial grid
    - ✓ Redistribute the nodes on the associated line into the SIAC
- Redistribute nodes on the external boundaries
- Smooth the overlapping mesh



The smoothing process seeks to relocate in a smooth manner the nodes in the overlapping mesh in order to get the quadrilateral mesh structured, conformed and tailored to the internal boundaries of the domain.

This is accomplished through the numerical solution of an elliptic partial differential equation based on finite differences.

Structured quadrilateral meshes

The transformation of the physical plane (x,y) (overlapping mesh) to the transformed plane (ξ,η) (smooth mesh) is given by the function

$$f(x,y) = (\xi,\eta), \text{ with } \xi = \xi(x,y),\ \eta = \eta(x,y).$$

Similarly, the inverse transformation is given by the functions $x = x(\xi, \eta)$, $y = y(\xi, \eta)$.

The partial derivatives of the function f respect to ξ and η are as follows:

$$\begin{pmatrix} f_\xi \\ f_\eta \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

Structured quadrilateral meshes

The smooth mesh satisfies the standard system of elliptic equations

$$\xi_{xx} + \xi_{yy} = 0$$
$$\eta_{xx} + \eta_{yy} = 0$$

with Dirichlet type boundary conditions, corresponding to the positions of fixed nodes on the internal and external boundaries.

These equations are analytically inverted and the calculations are carried out in the physical plane (overlapping mesh), being the non-linear inverted equations as follows

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0$$
$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0$$

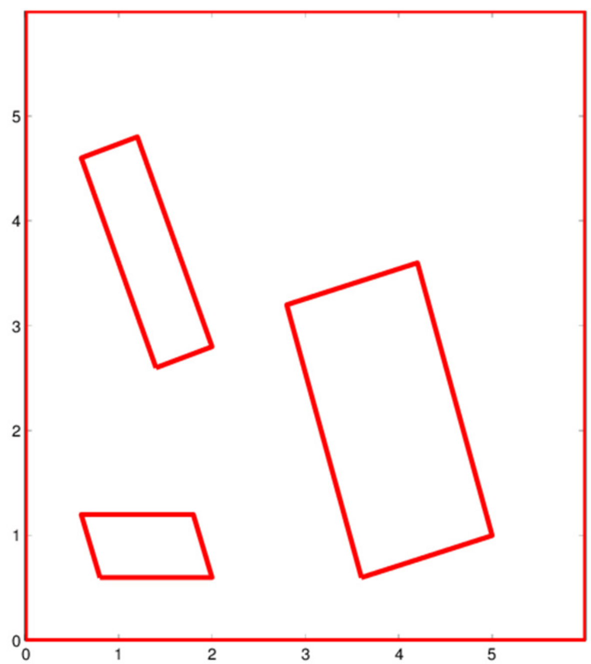
where $\alpha = x_{\eta}{}^{2} + y_{\eta}{}^{2}\,,\quad \beta = x_{\xi}x_{\eta} + y_{\xi}y_{\eta}\,,\quad \gamma = x_{\xi}{}^{2} + y_{\xi}{}^{2}\,.$

with transformed boundary conditions.

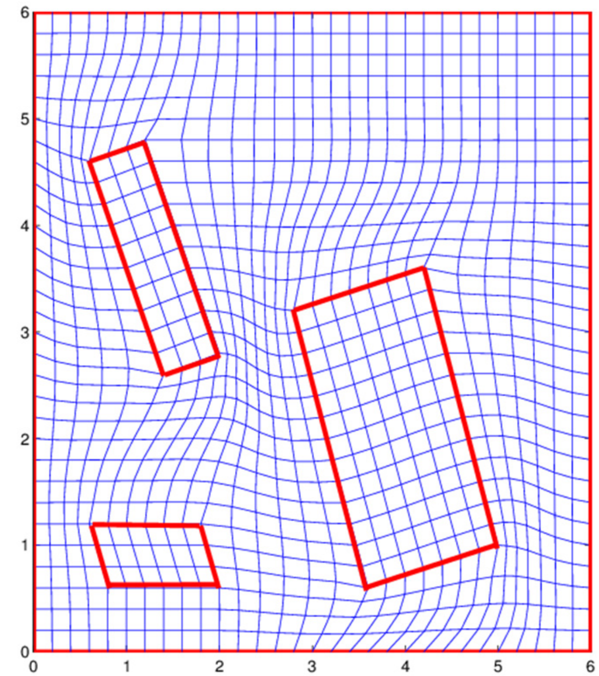
The elliptic partial differential equation will solved using finite differences.

Structured quadrilateral meshes

Example 1.



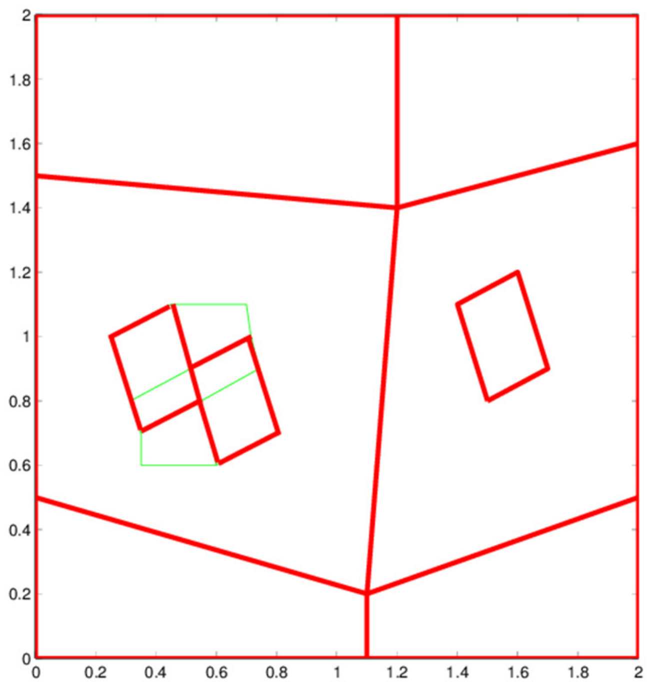
boundaries



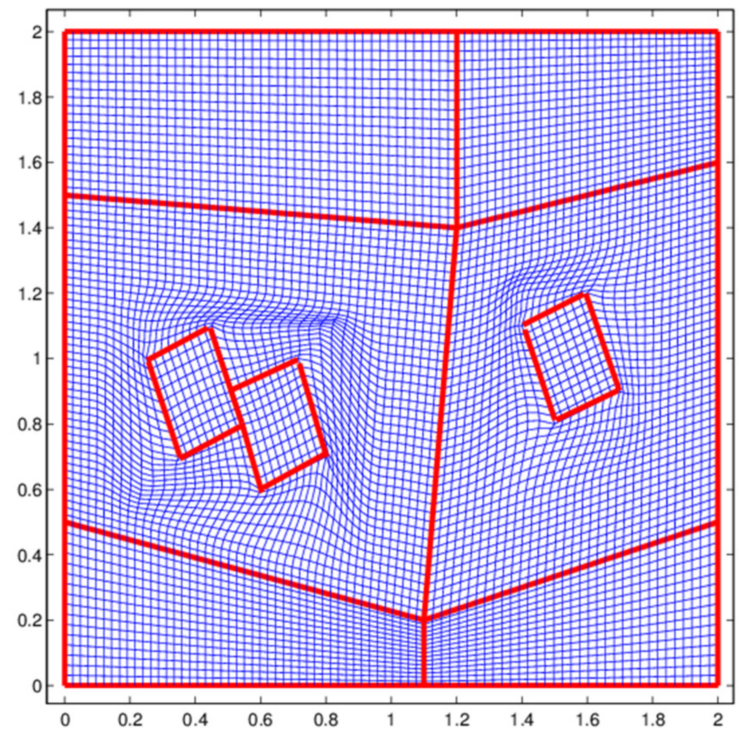
smooth mesh

Structured quadrilateral meshes

Example 2.



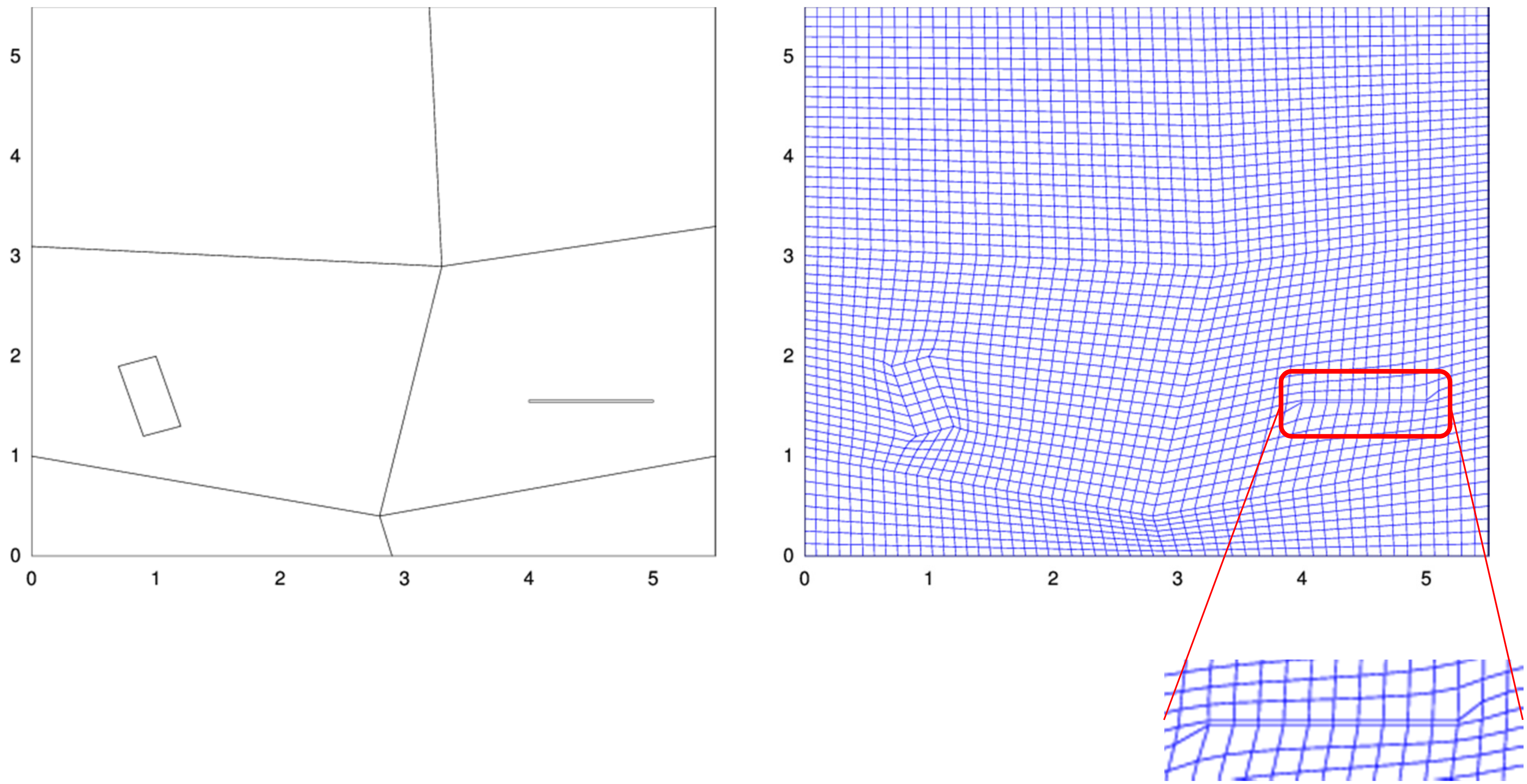
boundaries



smooth mesh

Structured quadrilateral meshes

Example 3.



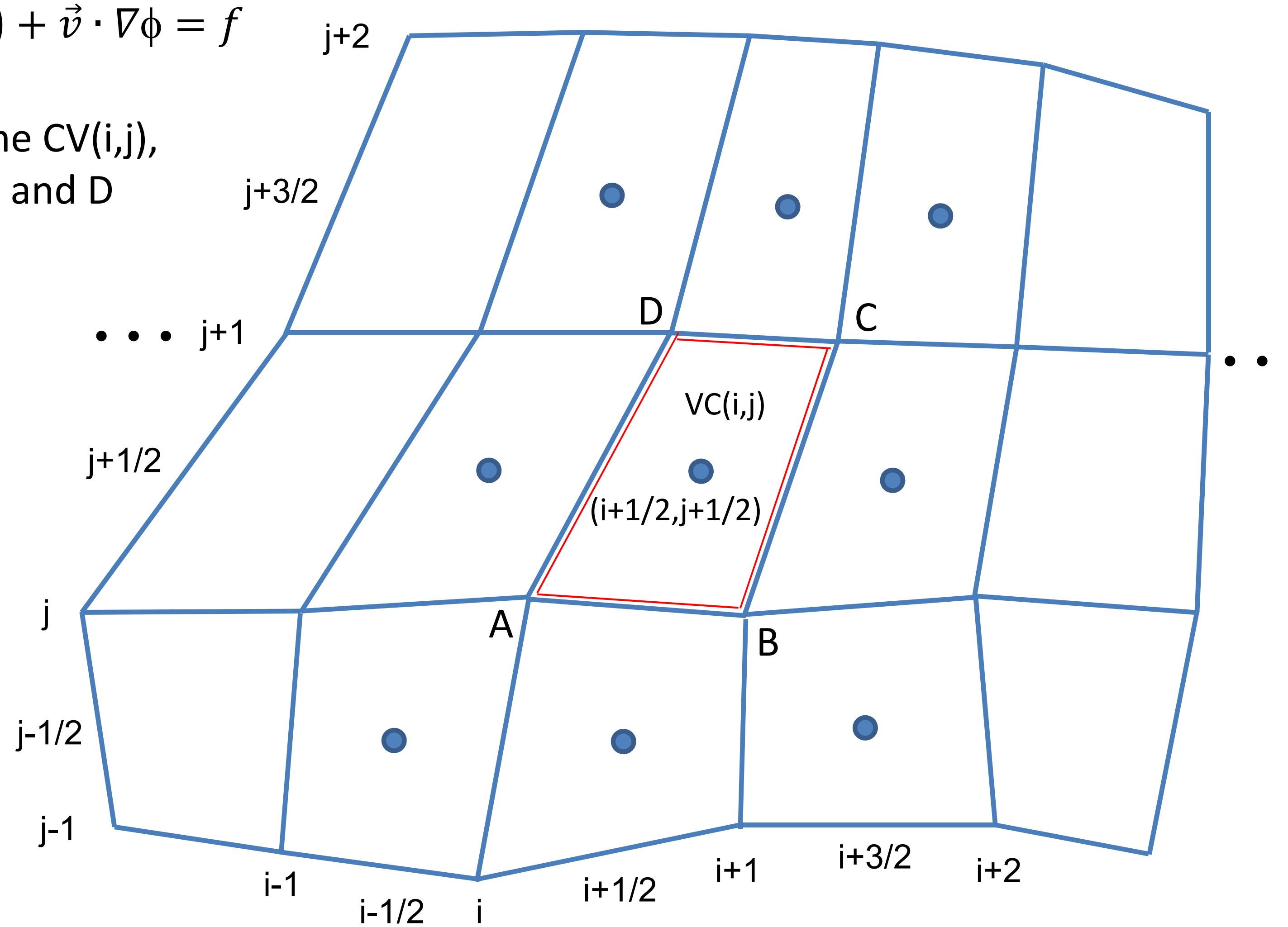
Numerical model

Steady state.

The convection diffusion equation

$$-\nabla \cdot \left(k\nabla\phi(\vec{r},t)\right) + \vec{v} \cdot \nabla\phi = f$$

The control volume CV(i,j), defined by A, B, C and D



Numerical model

Steady state.

Integrating the equation on CV(i,j) and applying the Green Theorem

$$-\oint_{ABCD} k\left(\frac{\partial\phi}{\partial x}dy-\frac{\partial\phi}{\partial y}dx\right)+\oint_{ABCD}\vec{v}\phi\cdot\vec{n}ds=\overline{f}_{i,j}S \qquad (1)$$

Approximating the integral of the first term of the equation (1) on the segments AB, BC, CD and DA

$$k\left\{\left[\frac{\partial\phi}{\partial x}\right]_{i+\frac{1}{2},j}\Delta y_{AB}-\left[\frac{\partial\phi}{\partial y}\right]_{i+\frac{1}{2},j}\Delta x_{AB}+\left[\frac{\partial\phi}{\partial x}\right]_{i+1,j+\frac{1}{2}}\Delta y_{BC}-\left[\frac{\partial\phi}{\partial y}\right]_{i+1,j+\frac{1}{2}}\Delta x_{BC}+\right.$$
$$\left.\left[\frac{\partial\phi}{\partial x}\right]_{i+\frac{1}{2},j+1}\Delta y_{CD}-\left[\frac{\partial\phi}{\partial y}\right]_{i+\frac{1}{2},j+1}\Delta x_{CD}+\left[\frac{\partial\phi}{\partial x}\right]_{i,j+\frac{1}{2}}\Delta y_{DA}-\left[\frac{\partial\phi}{\partial y}\right]_{i,j+\frac{1}{2}}\Delta x_{DA}\right\}$$

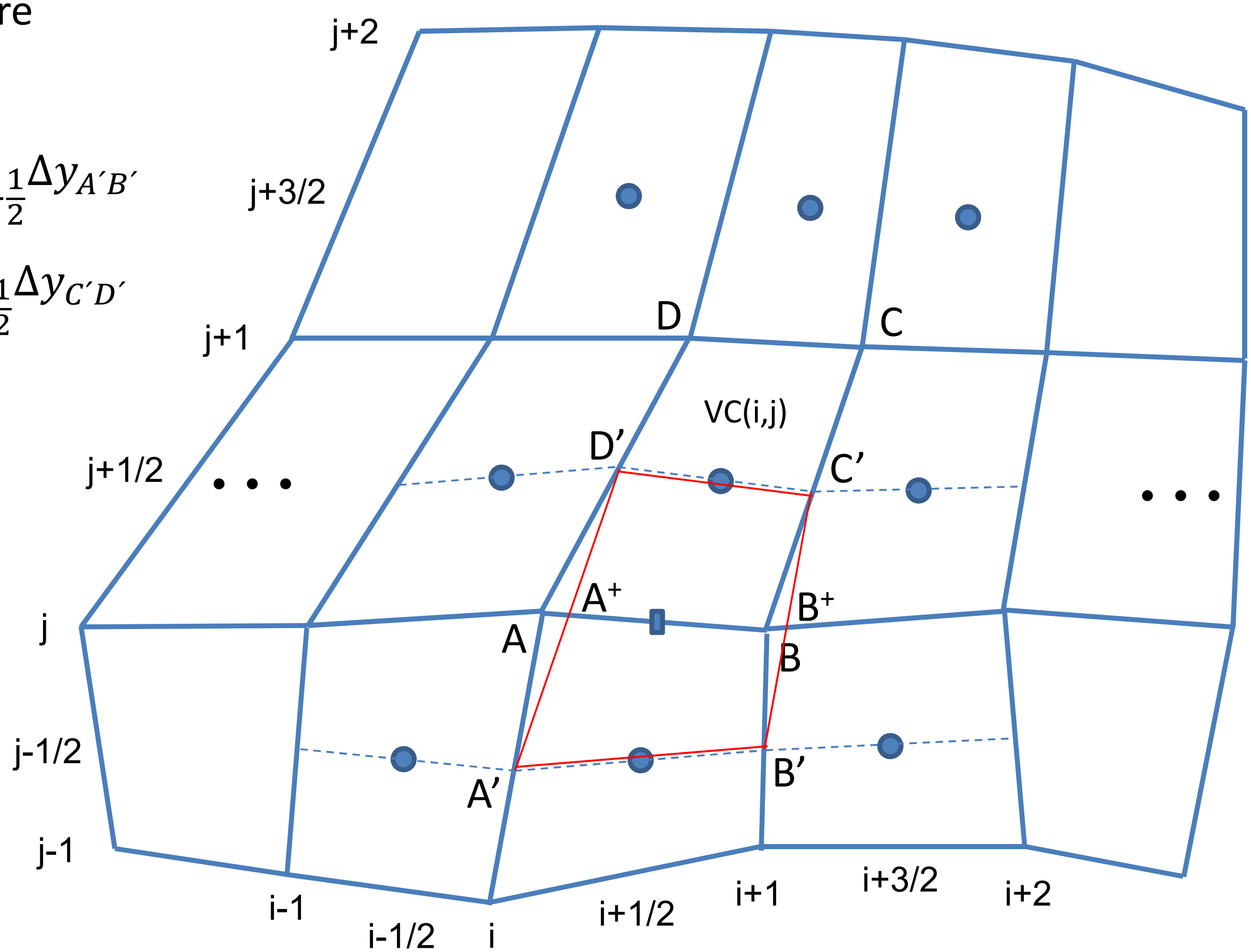
$$k=k_{i+\frac{1}{2},j+\frac{1}{2}}$$

Numerical model

Steady state.

To evaluate the partial derivatives in (i+½,j), will be used the average value on the area shown in the figure

$$\left[\frac{\partial\phi}{\partial x}\right]_{i+\frac{1}{2},j} \approx \frac{1}{S'}\,(\phi_{i+\frac{1}{2},j-\frac{1}{2}}\Delta y_{A'B'} + \phi_{B^+}\Delta y_{B'C'} + \phi_{i+\frac{1}{2},j+\frac{1}{2}}\Delta y_{C'D'} + \phi_{A^+}\Delta y_{D'A'})$$



Numerical model

Steady state.

Discrete expression for the first integral of the equation (1)

$$
\begin{aligned}
k\Bigg\{ & \frac{1}{4}(N_{DA}-N_{AB})\phi_{i-\frac{1}{2},j-\frac{1}{2}}+(M_{AB}-\frac{N_{BC}}{4}+\frac{N_{DA}}{4})\phi_{i+\frac{1}{2},j-\frac{1}{2}} \\
& +\frac{1}{4}(N_{AB}-N_{BC})\phi_{i+\frac{3}{2},j-\frac{1}{2}}+(M_{DA}+\frac{N_{CD}}{4}-\frac{N_{AB}}{4})\phi_{i-\frac{1}{2},j+\frac{1}{2}} \\
& -(M_{AB}+M_{BC}+M_{CD}+M_{DA})\phi_{i+\frac{1}{2},j+\frac{1}{2}} \\
& +(\frac{N_{AB}}{4}+M_{BC}-\frac{N_{CD}}{4})\phi_{i+\frac{3}{2},j+\frac{1}{2}}+\frac{1}{4}(N_{CD}-N_{DA})\phi_{i-\frac{1}{2},j+\frac{3}{2}} \\
& +(\frac{N_{BC}}{4}+M_{CD}-\frac{N_{DA}}{4})\phi_{i+\frac{1}{2},j+\frac{3}{2}}+\frac{1}{4}(N_{BC}-N_{CD})\phi_{i+\frac{3}{2},j+\frac{3}{2}}\Bigg\}
\end{aligned}
$$

where

$$M_{AB}=\frac{\Delta y_{AB}^2+\Delta x_{AB}^2}{S'} \qquad N_{AB}=\frac{\Delta y_{j-\frac{1}{2},j+\frac{1}{2}}\Delta y_{AB}+\Delta x_{j-\frac{1}{2},j+\frac{1}{2}}\Delta x_{AB}}{S'}$$

$$M_{BC}=\frac{\Delta y_{BC}^2+\Delta x_{BC}^2}{S''} \qquad N_{BC}=\frac{\Delta y_{i+\frac{3}{2},i+\frac{1}{2}}\Delta y_{BC}+\Delta x_{i+\frac{3}{2},i+\frac{1}{2}}\Delta x_{BC}}{S''}$$

$$M_{CD}=\frac{\Delta y_{CD}^2+\Delta x_{CD}^2}{S^*} \qquad N_{CD}=\frac{\Delta y_{j+\frac{3}{2},j+\frac{1}{2}}\Delta y_{CD}+\Delta x_{j+\frac{3}{2},j+\frac{1}{2}}\Delta x_{CD}}{S^*}$$

$$M_{DA}=\frac{\Delta y_{DA}^2+\Delta x_{DA}^2}{S^{**}} \qquad N_{DA}=\frac{\Delta y_{i-\frac{1}{2},i+\frac{1}{2}}\Delta y_{DA}+\Delta x_{i-\frac{1}{2},i+\frac{1}{2}}\Delta x_{DA}}{S^{**}}$$

Numerical model

Steady state.

The second integral in the equation (1)

$$-\oint_{ABCD} k\left(\frac{\partial\phi}{\partial x}dy - \frac{\partial\phi}{\partial y}dx\right) + \oint_{ABCD} \vec{v}\phi\cdot\vec{n}ds = \overline{f}_{i,j}S$$

is approximated on the segments AB, BC, CD and DA by

$$\begin{aligned}
&[\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j}[\phi]_{i+\frac{1}{2},j}\Delta y_{AB} + [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j}[\phi]_{i+\frac{1}{2},j}\Delta x_{AB} \\
&+[\vec{v}\cdot\vec{n}]_{i+1,j+\frac{1}{2}}[\phi]_{i+1,j+\frac{1}{2}}\Delta y_{BC} + [\vec{v}\cdot\vec{n}]_{i+1,j+\frac{1}{2}}[\phi]_{i+1,j+\frac{1}{2}}\Delta x_{BC} \\
&+[\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j+1}[\phi]_{i+\frac{1}{2},j+1}\Delta y_{CD} + [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j+1}[\phi]_{i+\frac{1}{2},j+1}\Delta x_{CD} \\
&+[\vec{v}\cdot\vec{n}]_{i,j+\frac{1}{2}}[\phi]_{i,j+\frac{1}{2}}\Delta y_{DA} + [\vec{v}\cdot\vec{n}]_{i,j+\frac{1}{2}}[\phi]_{i,j+\frac{1}{2}}\Delta x_{DA}
\end{aligned}$$

Upwind approximation for ϕ in (i+½,j)

$$[\phi]_{i+\frac{1}{2},j} = \beta_{AB}\phi_{i+\frac{1}{2},j-\frac{1}{2}} + (1-\beta_{AB})\phi_{i+\frac{1}{2},j+\frac{1}{2}}$$

with $$\beta = \begin{cases} 1 \;\; if\; [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j} \geq 0 \\ 0 \;\; if\; [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j} < 0 \end{cases}$$

Numerical model

Steady state.

Discrete expression for the second integral of the equation (1)

$$
\begin{aligned}
&(\beta_{AB}\, O_{AB})\phi_{i+\frac{1}{2},j-\frac{1}{2}} + (\beta_{DA}\, O_{DA})\phi_{i-\frac{1}{2},j+\frac{1}{2}} \\
\{(1-\beta_{AB})O_{AB} + (1-\beta_{BC})O_{BC} + (1-\beta_{CD})O_{CD} + (1-\beta_{DA})O_{DA}\}&\phi_{i+\frac{1}{2},j+\frac{1}{2}} \\
&(\beta_{BC}\, O_{BC})\phi_{i+\frac{3}{2},j+\frac{1}{2}} + (\beta_{CD}\, O_{CD})\phi_{i+\frac{1}{2},j+\frac{3}{2}}
\end{aligned}
$$

where

$$
\begin{aligned}
O_{AB} &= [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j}\,(\Delta y_{AB} + \Delta x_{AB}) \\
O_{BC} &= [\vec{v}\cdot\vec{n}]_{i+1,j+\frac{1}{2}}\,(\Delta y_{BC} + \Delta x_{BC}) \\
O_{CD} &= [\vec{v}\cdot\vec{n}]_{i+\frac{1}{2},j+1}\,(\Delta y_{CD} + \Delta x_{CD}) \\
O_{DA} &= [\vec{v}\cdot\vec{n}]_{i,j+\frac{1}{2}}\,(\Delta y_{DA} + \Delta x_{DA})
\end{aligned}
$$

Numerical model

Steady state.

Finally the discrete expression for the convection diffusion equation (1)

$$
\begin{aligned}
-\psi = {} & \frac{k}{4}(N_{AB}-N_{DA})\phi_{i-\frac{1}{2},j-\frac{1}{2}} + \{k(-M_{AB}+\frac{N_{BC}}{4}-\frac{N_{DA}}{4})+\beta_{AB}O_{AB}\}\phi_{i+\frac{1}{2},j-\frac{1}{2}} \\
& +\frac{k}{4}(-N_{AB}+N_{BC})\phi_{i+\frac{3}{2},j-\frac{1}{2}} + \{k(-M_{DA}-\frac{N_{CD}}{4}+\frac{N_{AB}}{4})+\beta_{DA}O_{DA}\}\phi_{i-\frac{1}{2},j+\frac{1}{2}} \\
& +\{k(M_{AB}+M_{BC}+M_{CD}+M_{DA}) \\
& +(1-\beta_{AB})O_{AB}+(1-\beta_{BC})O_{BC}+(1-\beta_{CD})O_{CD}+(1-\beta_{DA})O_{DA}\}\phi_{i+\frac{1}{2},j+\frac{1}{2}} \\
& +\{k(-\frac{N_{AB}}{4}-M_{BC}+\frac{N_{CD}}{4})+\beta_{BC}O_{BC}\}\phi_{i+\frac{3}{2},j+\frac{1}{2}} + \frac{k}{4}(-N_{CD}+N_{DA})\phi_{i-\frac{1}{2},j+\frac{3}{2}} \\
& +\{k(-\frac{N_{BC}}{4}-M_{CD}+\frac{N_{DA}}{4})+\beta_{CD}O_{CD}\}\phi_{i+\frac{1}{2},j+\frac{3}{2}} + \frac{k}{4}(-N_{BC}+N_{CD})\phi_{i+\frac{3}{2},j+\frac{3}{2}} \\
& -\overline{f}_{i+\frac{1}{2},j+\frac{1}{2}}S = 0
\end{aligned}
$$

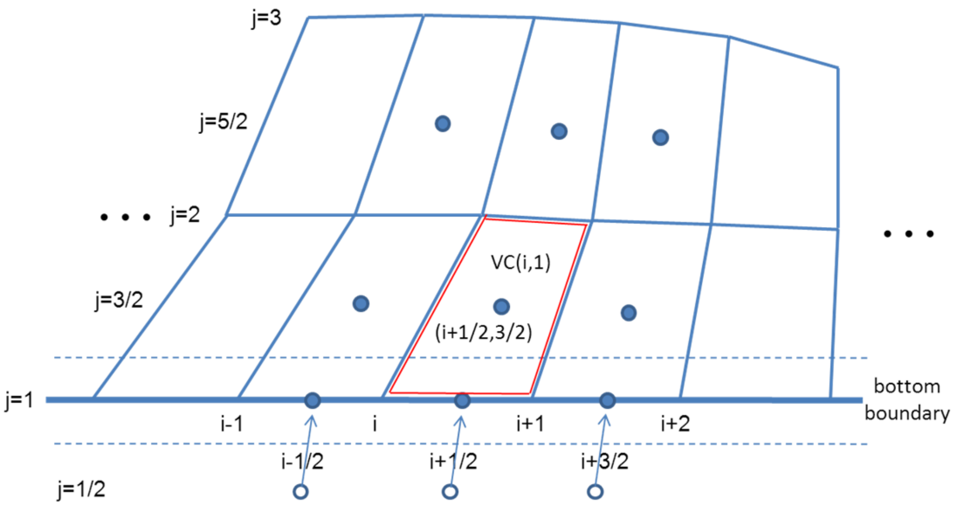
where $k = k_{i+\frac{1}{2},j+\frac{1}{2}}$

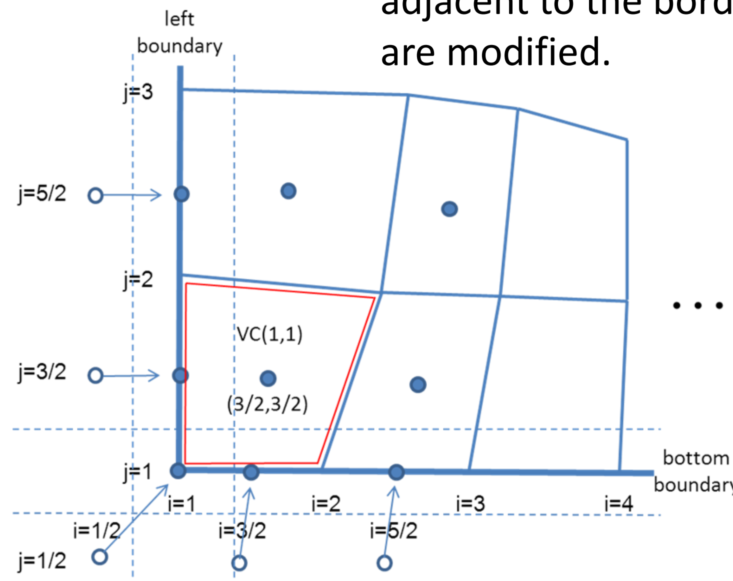
Numerical model

Boundary conditions.

Dirichlet conditions.

Control volumes adjacent to the border are modified.





Newman conditions.

$$\left[\frac{\partial \phi}{\partial y}\right]_{i+\frac{1}{2},1} = h_{inf\, i+\frac{1}{2}}$$ left boundary

$$\left[\frac{\partial \phi}{\partial y}\right]_{i+\frac{1}{2},n} = h_{sup\, i+\frac{1}{2}}$$ top boundary

$$\left[\frac{\partial \phi}{\partial x}\right]_{1,j+\frac{1}{2}} = h_{izq\, j+\frac{1}{2}}$$ right boundary

$$\left[\frac{\partial \phi}{\partial x}\right]_{n,j+\frac{1}{2}} = h_{der\, j+\frac{1}{2}}$$ bottom boundary

Numerical model

Transient state.

Convection diffusion equation including the term varying in time

$$\frac{\partial \phi}{\partial t} = k\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) - \vec{v}\cdot\nabla\phi + f$$

Integrating on the Control Volume ABCD

$$\iint_{ABCD} \frac{\partial \phi}{\partial t} dA = \iint_{ABCD} \left(k\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) - \vec{v}\cdot\nabla\phi + f\right) dA$$

The integral on the right hand side was already calculated and denoted by Ψ.
Integrating in time between t_0 and t_1

$$\iint_{ABCD} \int_{t_0}^{t_1} \frac{\partial \phi}{\partial t} dt\, dA = \int_{t_0}^{t_1} \psi dt \quad \Rightarrow \quad \iint_{ABCD} (\phi^1 - \phi^0) dA = \int_{t_0}^{t_1} \psi dt$$

Considering a convex combination for the right hand side follows the implicit method

$$\phi^1_{i+\frac{1}{2},j+\frac{1}{2}} - \frac{\Delta t}{S}\psi^1 = \phi^0_{i+\frac{1}{2},j+\frac{1}{2}}$$

Numerical model

Transient state.

Finally, the discrete system is as follows

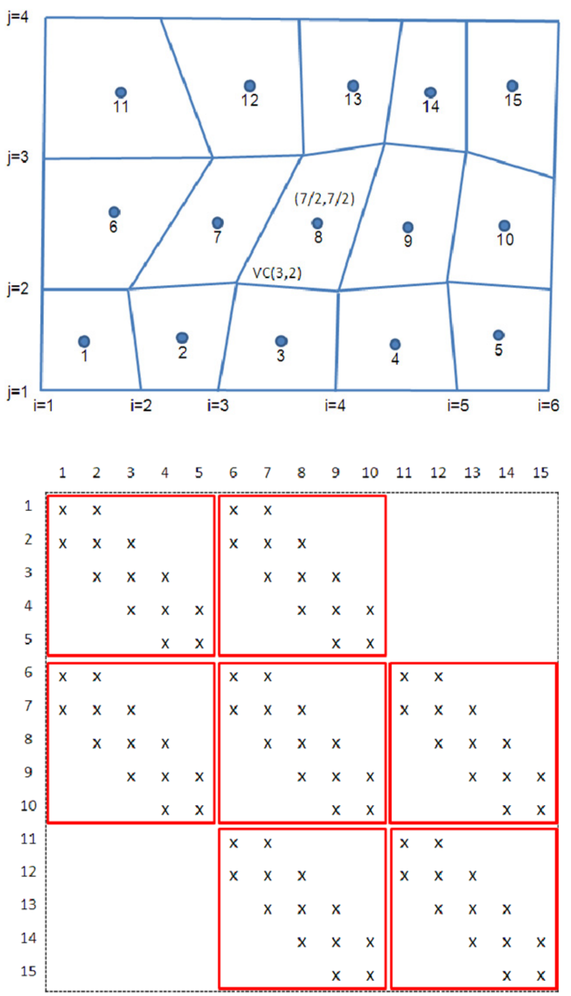
$$
\begin{aligned}
&\frac{\Delta t}{S}\frac{k}{4}(N_{AB}-N_{DA})\phi^1_{i-\frac{1}{2},j-\frac{1}{2}}+\frac{\Delta t}{S}\{k(-M_{AB}+\frac{N_{BC}}{4}-\frac{N_{DA}}{4})+\beta_{AB}O_{AB}\}\phi^1_{i+\frac{1}{2},j-\frac{1}{2}}\\
&+\frac{\Delta t}{S}\frac{k}{4}(-N_{AB}+N_{BC})\phi^1_{i+\frac{3}{2},j-\frac{1}{2}}+\frac{\Delta t}{S}\{k(-M_{DA}-\frac{N_{CD}}{4}+\frac{N_{AB}}{4})+\beta_{DA}O_{DA}\}\phi^1_{i-\frac{1}{2},j+\frac{1}{2}}\\
&+(1+\frac{\Delta t}{S}\{k(M_{AB}+M_{BC}+M_{CD}+M_{DA})\\
&+(1-\beta_{AB})O_{AB}+(1-\beta_{BC})O_{BC}+(1-\beta_{CD})O_{CD}+(1-\beta_{DA})O_{DA}\})\phi^1_{i+\frac{1}{2},j+\frac{1}{2}}\\
&+\frac{\Delta t}{S}\{k(-\frac{N_{AB}}{4}-M_{BC}+\frac{N_{CD}}{4})+\beta_{BC}O_{BC}\}\phi^1_{i+\frac{3}{2},j+\frac{1}{2}}+\frac{\Delta t}{S}\frac{k}{4}(-N_{CD}+N_{DA})\phi^1_{i-\frac{1}{2},j+\frac{3}{2}}\\
&+\frac{\Delta t}{S}\{k(-\frac{N_{BC}}{4}-M_{CD}+\frac{N_{DA}}{4})+\beta_{CD}O_{CD}\}\phi^1_{i+\frac{1}{2},j+\frac{3}{2}}+\frac{\Delta t}{S}\frac{k}{4}(-N_{BC}+N_{CD})\phi^1_{i+\frac{3}{2},j+\frac{3}{2}}\\
&=\phi^0_{i+\frac{1}{2},j+\frac{1}{2}}+\Delta t\,\overline{f}_{i+\frac{1}{2},j+\frac{1}{2}}
\end{aligned}
$$

with some variant for de CV on the boundary depending on the boundary conditions (Dirichlet or Newman).

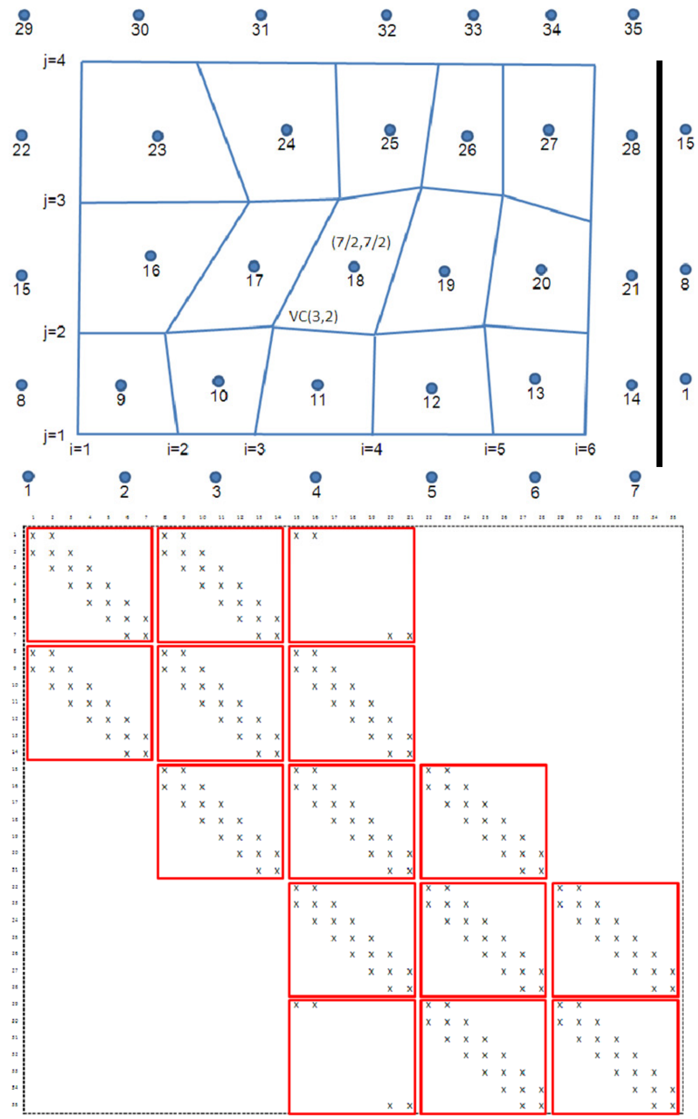
The resultant linear system, sparse , generally non-symmetric and ill-conditioned, and large dimensioned, will be solved using GMRES with incomplete LU preconditioning.

Numerical model

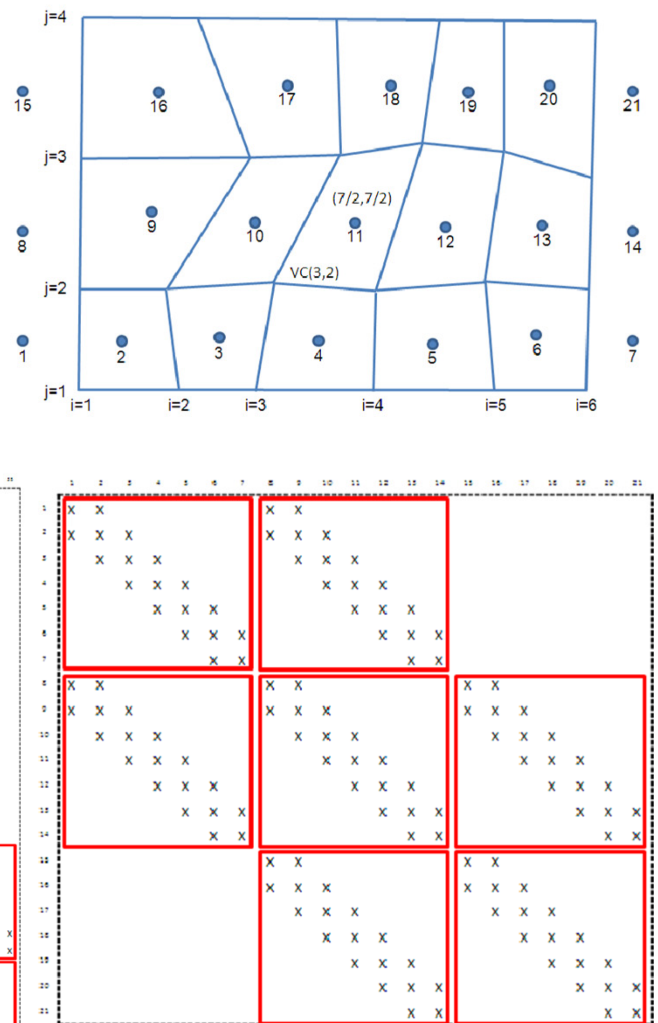
Structure of the linear system.



Dirichlet boundary conditions



Newman boundary conditions

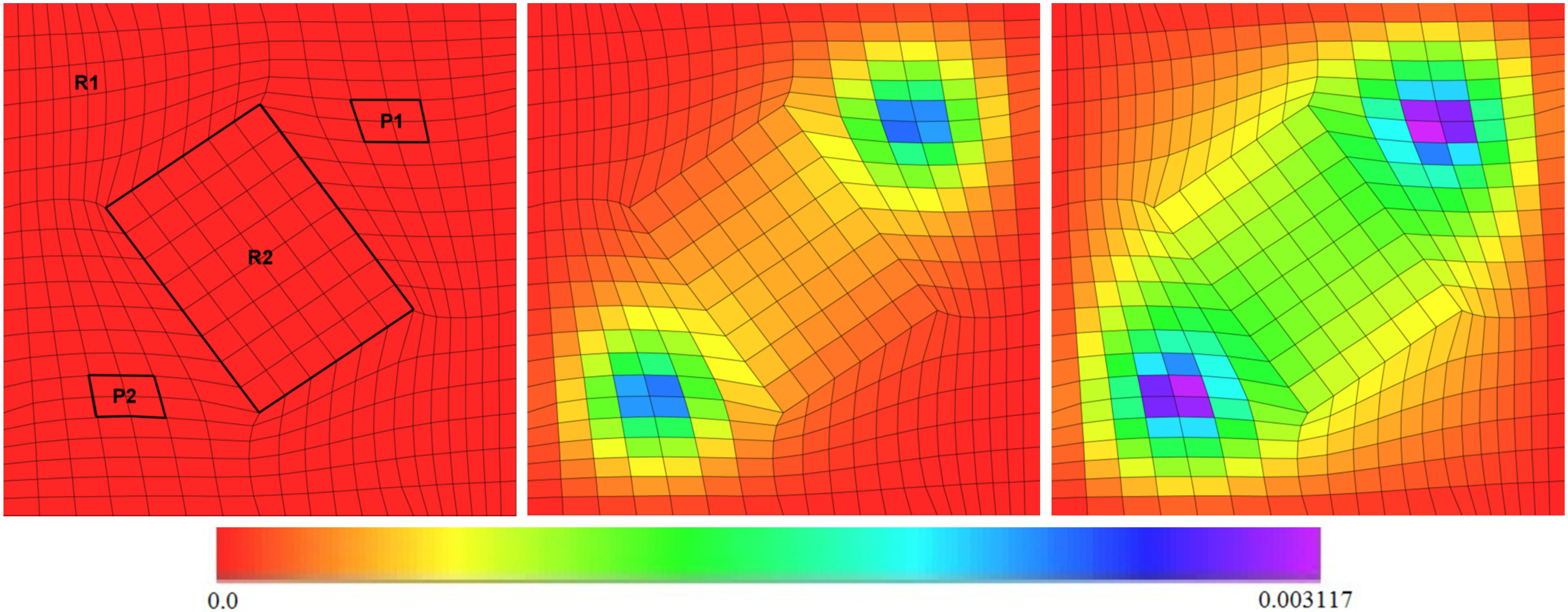


Mixed boundary conditions

Results

Example 1.

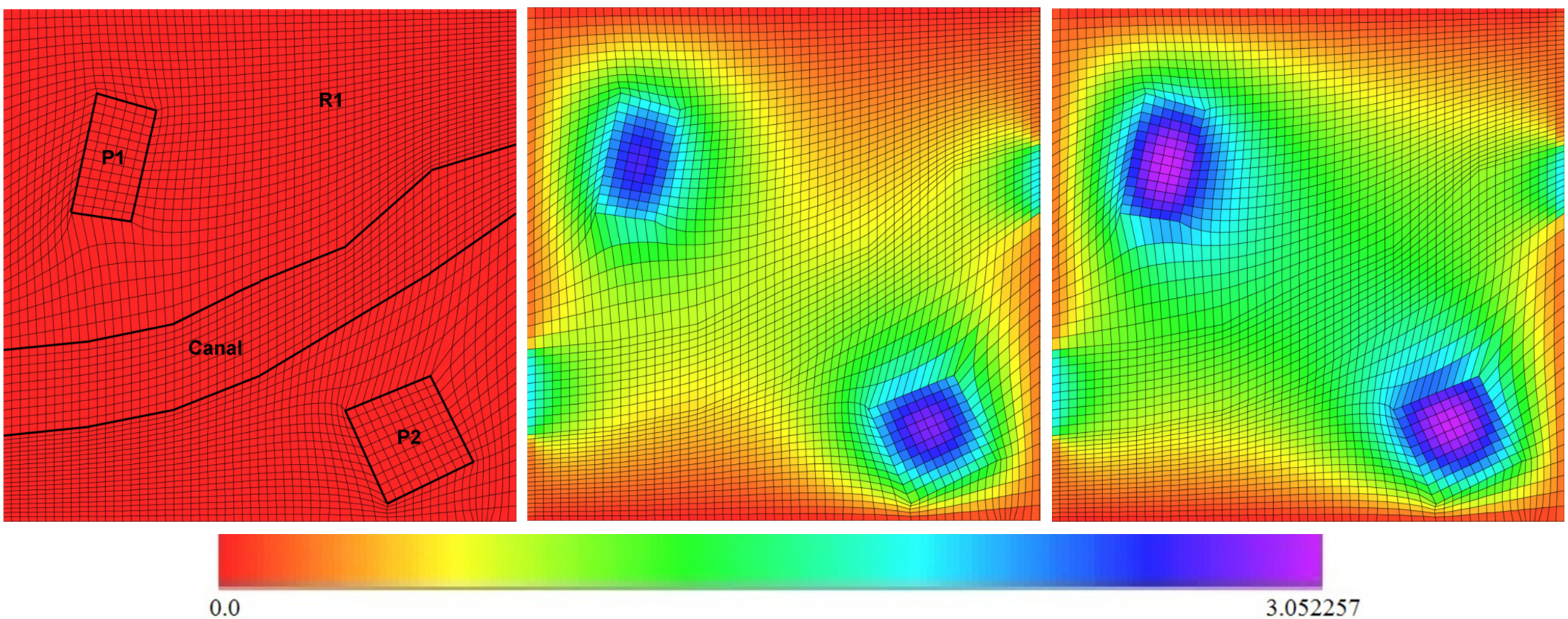
k = 2 in R2, k = 1 en R1. Dirichlet type boundary conditions with zero value on the boundary. Sources in P1 and P2 with f = 300.



Results

Example 2.

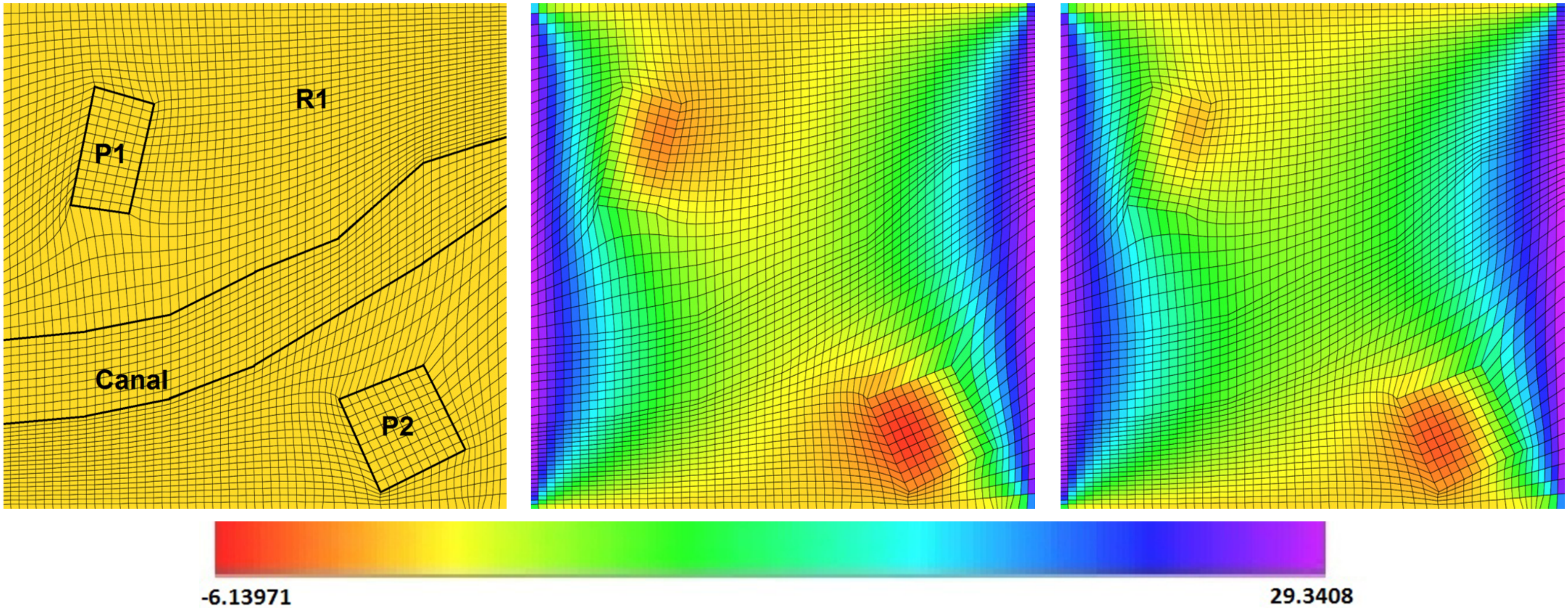
k = 200 in Canal and k = 5 in P1 and P2. Dirichlet type boundary conditions with zero value on the top and bottom boundary, 0.2 on the left and right boundary except on the zone corresponding to the “Canal” where is 2. Sources in P1 and P2 with f = 50.



Results

Example 3.

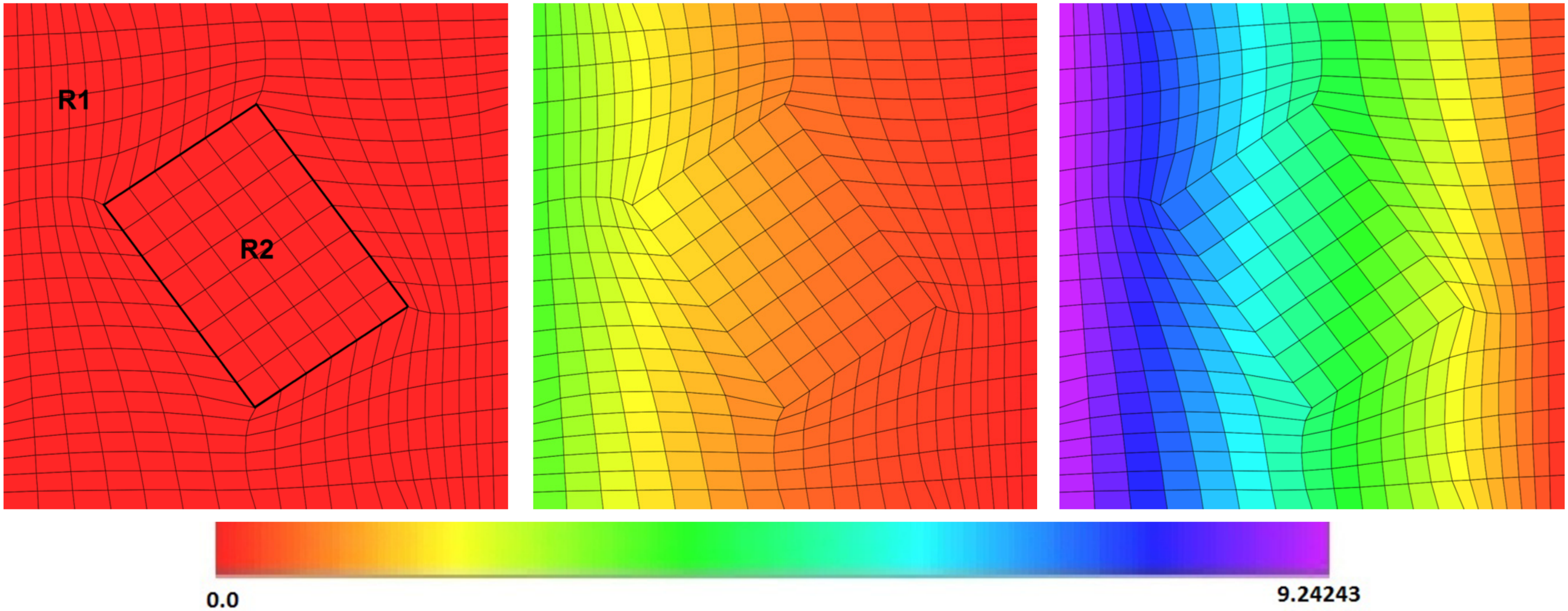
k = 1000 in Canal and k = 1 in R1. Dirichlet type boundary conditions with zero value on the top and bottom boundary, 30 on the left and right boundary. Sinks in P1 and P2 with f = -50.



Results

Example 4.

k = 1000 in R2 and k = 1 in R1. Mixed boundary conditions: right boundary ϕ=0, left boundary $\partial\phi/\partial n$=10, top and bottom boundaries $\partial\phi/\partial n$=0.



Conclusions

- A numerical model based on the finite volume methods for the solution of the convection diffusion equation in 2D was developed.
- The physical domain was discretized using non-rectangular grids formed only by quadrilaterals honoring the internal structures of a reservoir.
- The numerical model was implemented for both steady and unsteady state using modular programming in C language.
- Different scenarios were considered varying boundary conditions (Dirichlet and Neumann type), source and sink term, and diffusion constant fluid velocity. The results are consistent with the physical interpretation of each configuration.

Upcoming

- configurations involving complex structures/geometries
- boundary conditions on internal and external boundaries
- two-dimensional three-phase black oil fluid flow equations

⋮

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An up-wind finite volume method on non-orthogonal quadrilateral meshes for the convection diffusion equation in a porous media

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Thank for your attention